

Stack & Queue ADTs



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Today's Plan



ADT Recap

Stack ADT

Stack Applications

Queue ADT

Queue Applications

ADT Recap

Abstract Data Types:

Bag (unordered)

List (ordered)

ADT operations

add/insert, remove, find

Stack

34

An ADT representing a stack of items

Objects can be **pushed** onto the stack or **popped** from the stack

Stack

An ADT representing a stack of items

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34

Stack

An ADT representing a stack of items

Objects can be **pushed** onto the stack or **popped** from the stack



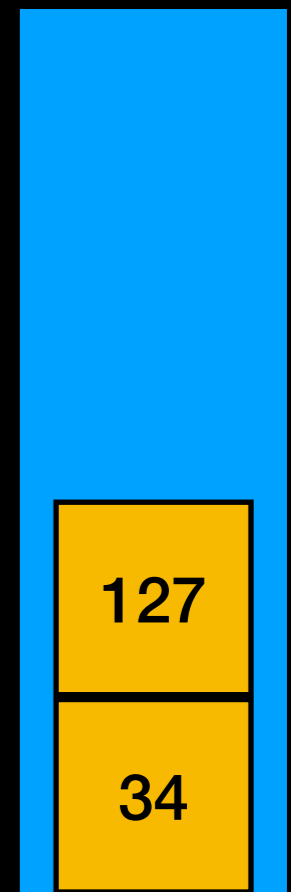
127

34

Stack

An ADT representing a stack of items

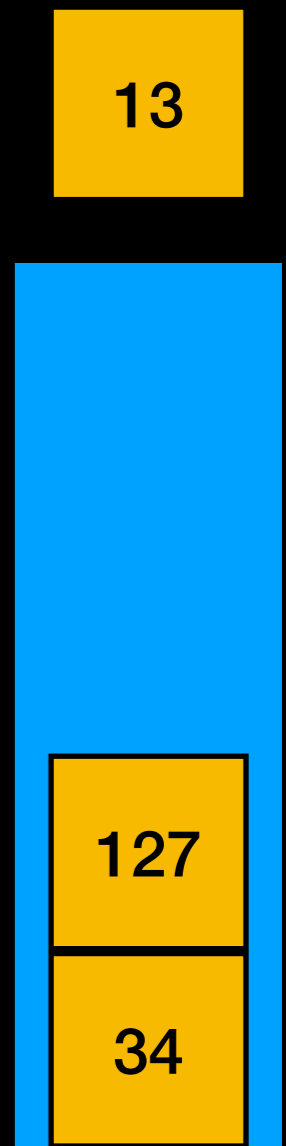
Objects can be **pushed** onto the stack or **popped** from the stack



Stack

An ADT representing a stack of items

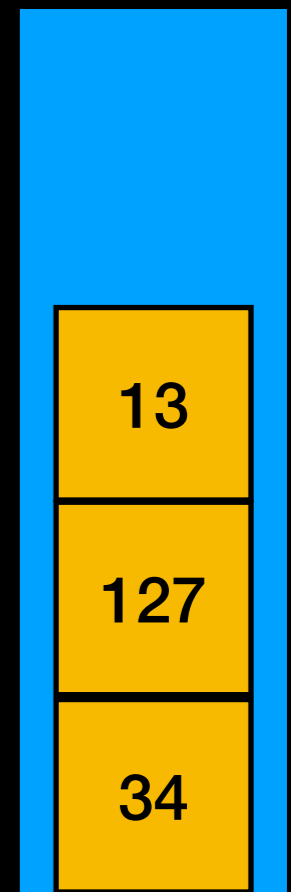
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Stack

An ADT representing a stack of items

Objects can be **pushed** onto the stack or **popped** from the stack



Stack

13

An ADT representing a stack of items

Objects can be **pushed** onto the stack or **popped** from the stack

127

34

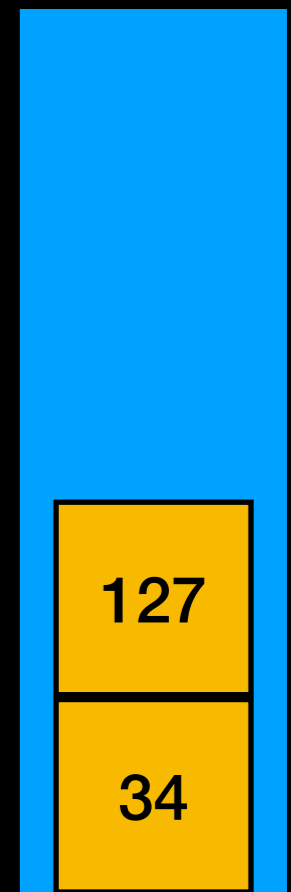
Stack

An ADT representing a stack of items

Objects can be **pushed** onto the stack or **popped** from the stack

LIFO: Last In First Out

Only top of stack is accessible (**top**), no other objects on the stack are visible



Applications

Very simple structure

Many applications:

- program stack

- balancing parenthesis

- evaluating postfix expressions

- backtracking

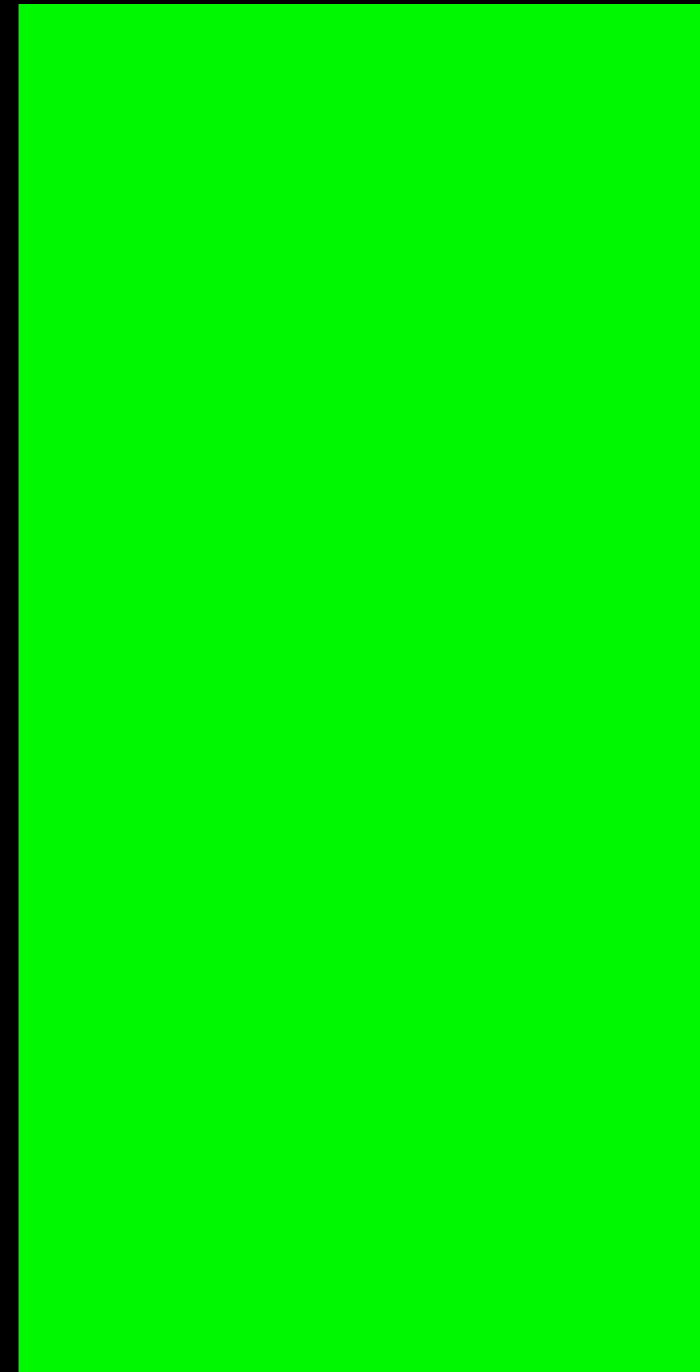
- ... and more

Program Stack

Program Stack

```
1 void f(int x, int y)
2 {
3     int a;
4     // stuff here
5     if(a<13)
6         a = g(a);
7     // stuff here
8 }

9 int g(int z)
10 {
11     int p ,q;
12     // stuff here
13     return q;
14 }
```

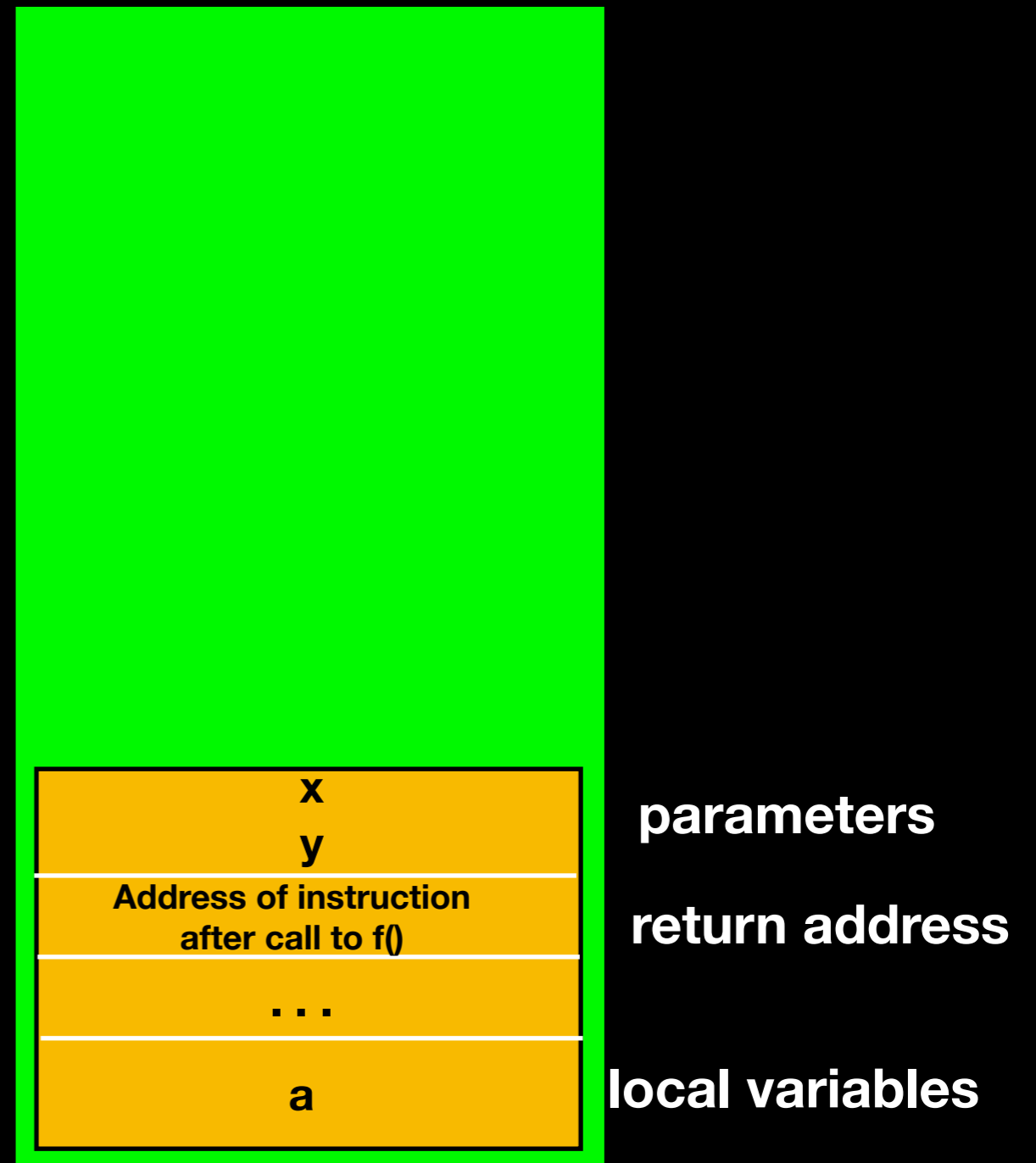


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**Stack Frame
for f()**



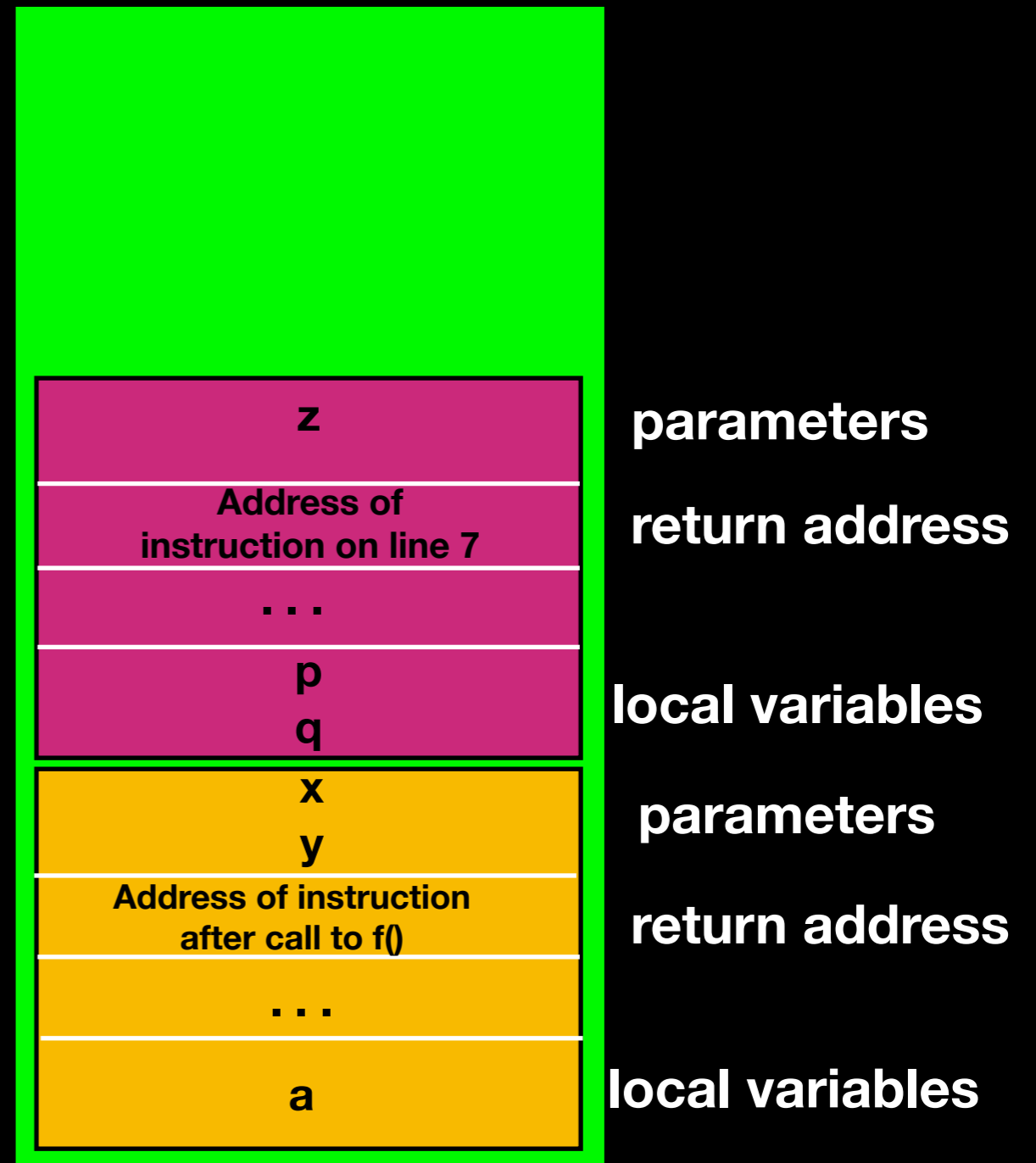
Program Stack

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5     if(a<13)
6         a = g(a);
7     // stuff here
8 }
```

**Stack Frame
for g()**

```
9 int g(int z)
10 {
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12     // stuff here
13     return q;
14 }
```

**Stack Frame
for f()**

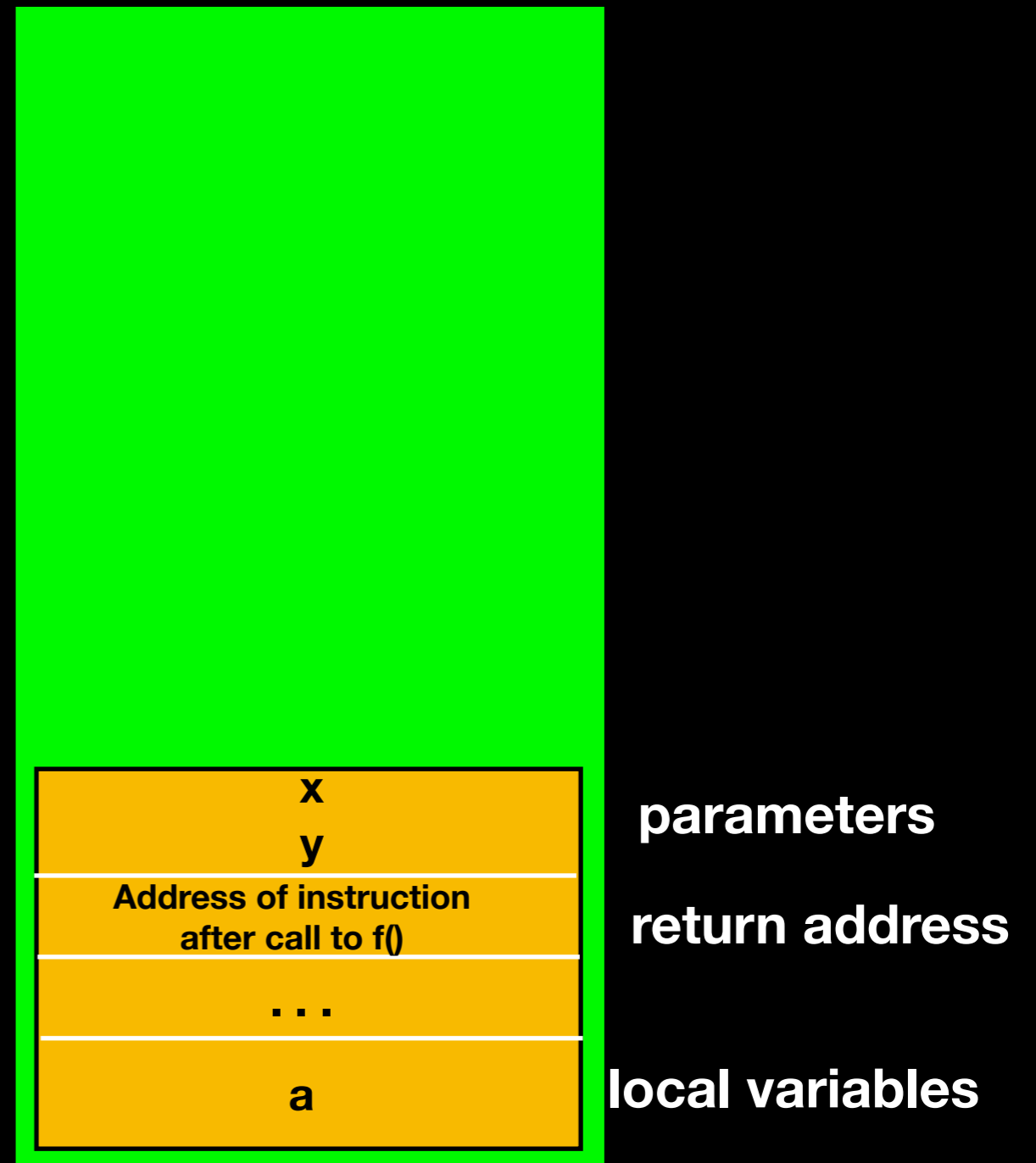


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**Stack Frame
for f()**



How would you solve it?

Balancing Parentheses

Given a string, determine if parentheses are balanced.

Parentheses can be {}, [], or (), and must be nested properly.

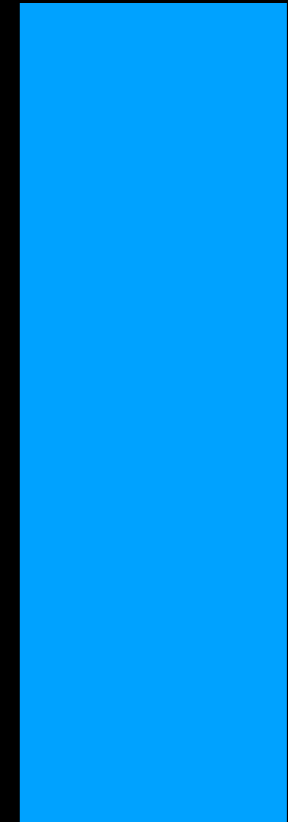
E.g. "[({ })]" is balanced, while "[({ }]" or "[({) }]" are not.

Typical applications: parsers and compilers.

Balancing Parentheses

```
int f() {if(x*(y+z[i])<47) {x += y}}
```

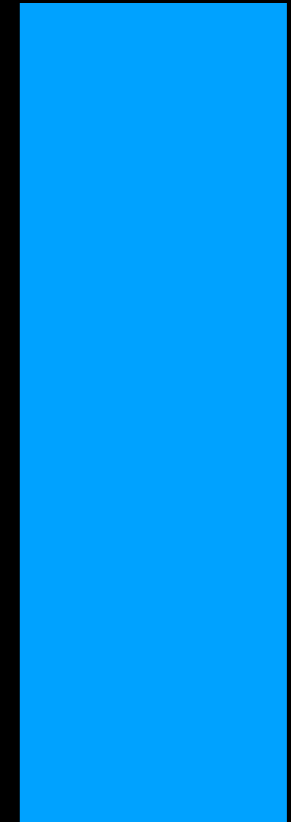
↑



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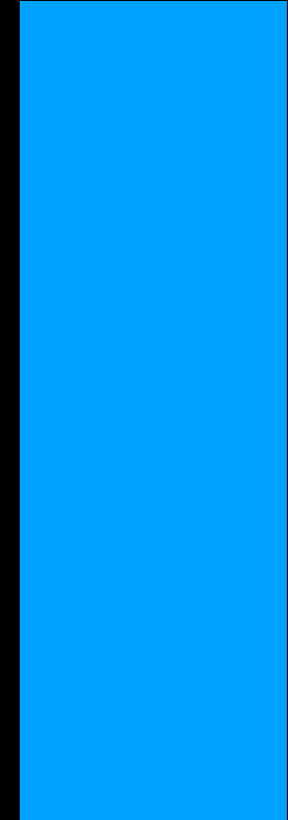
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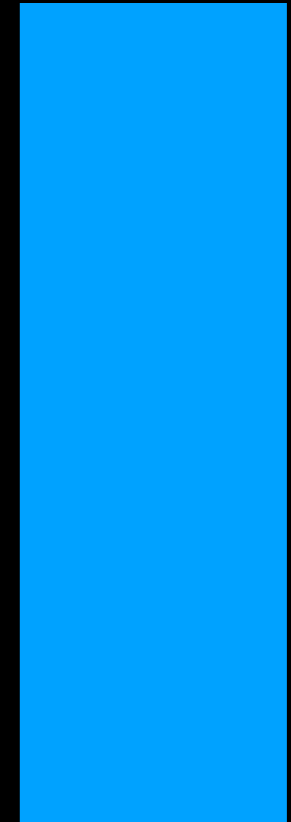

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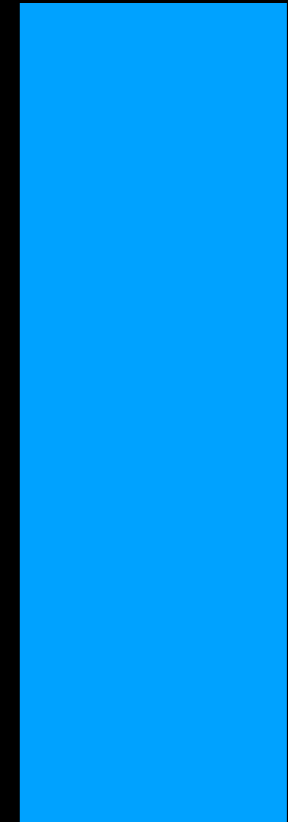
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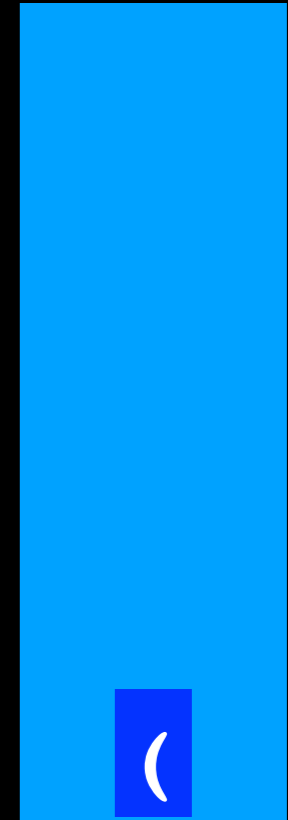
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Balancing Parentheses

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push

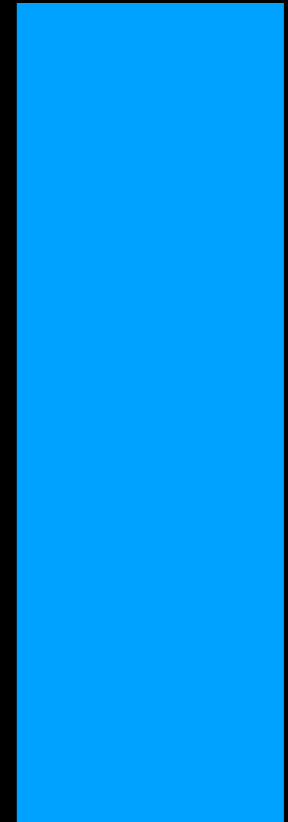


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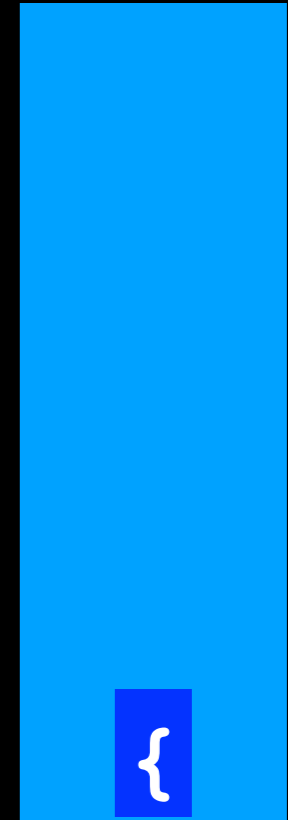
pop



Balancing Parentheses

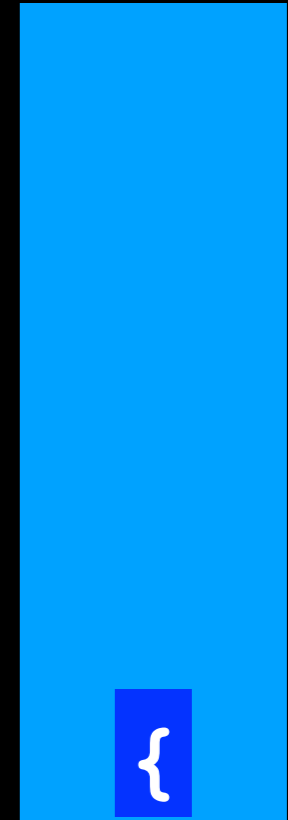
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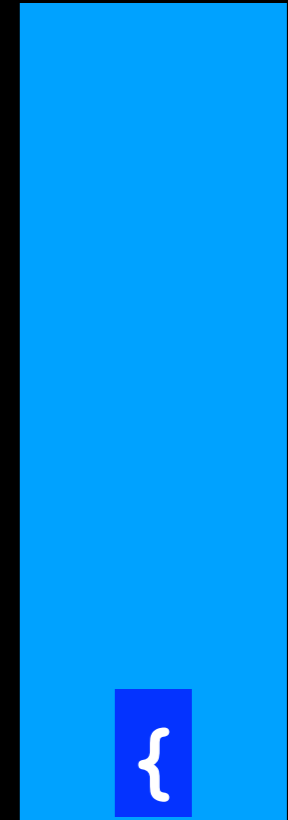
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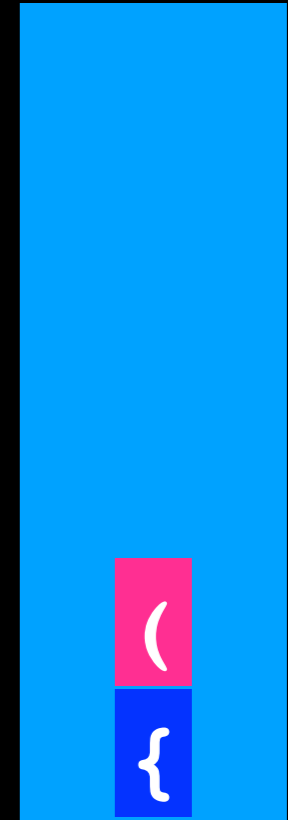
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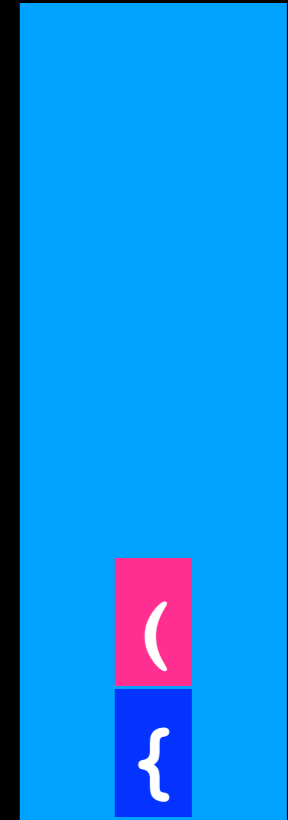
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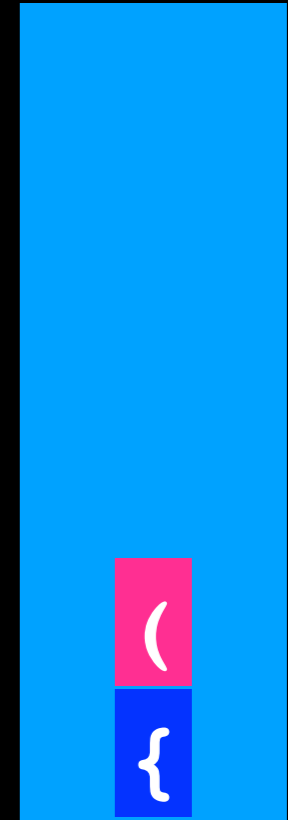
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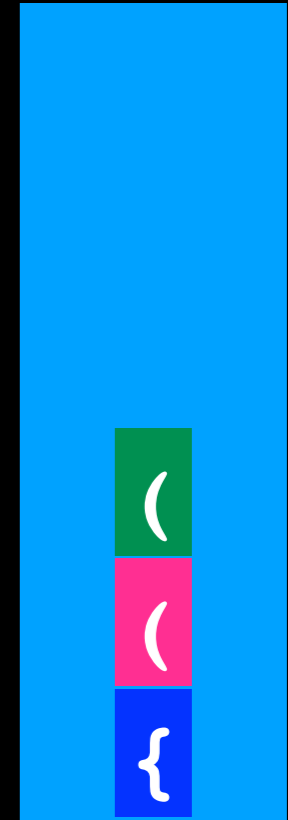
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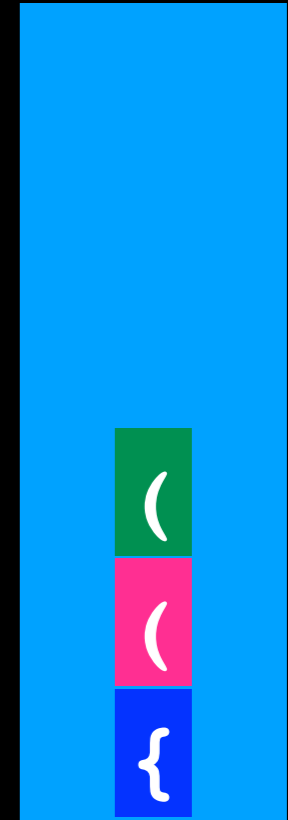
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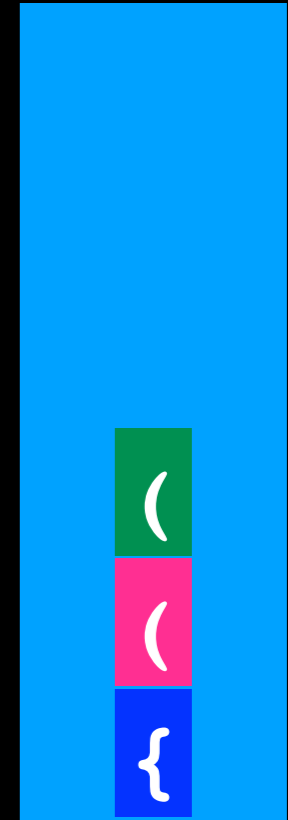
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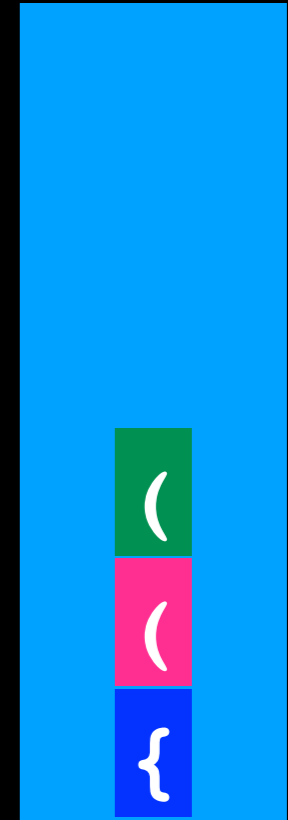
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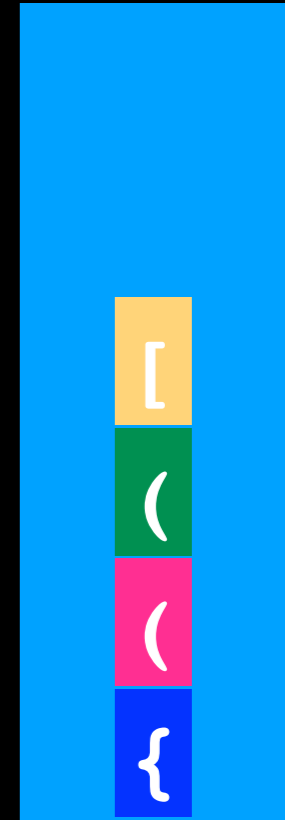


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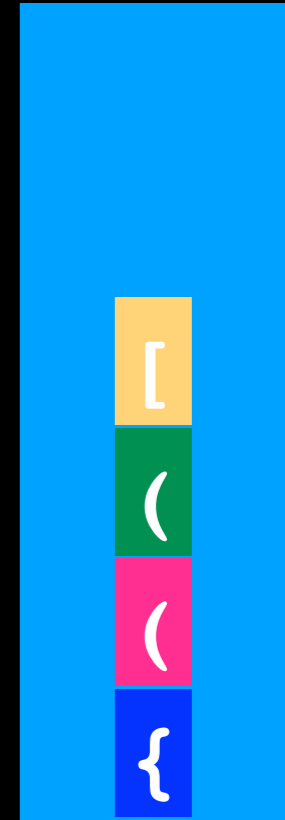


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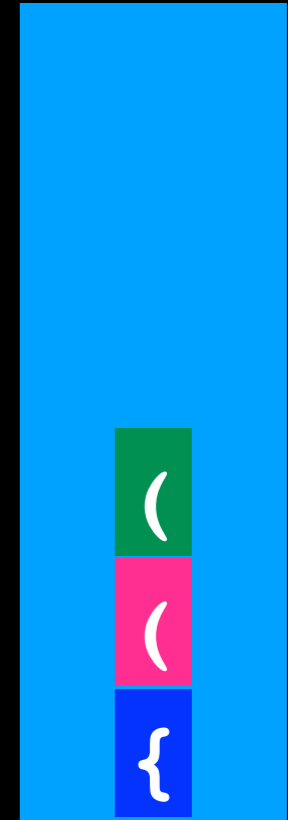
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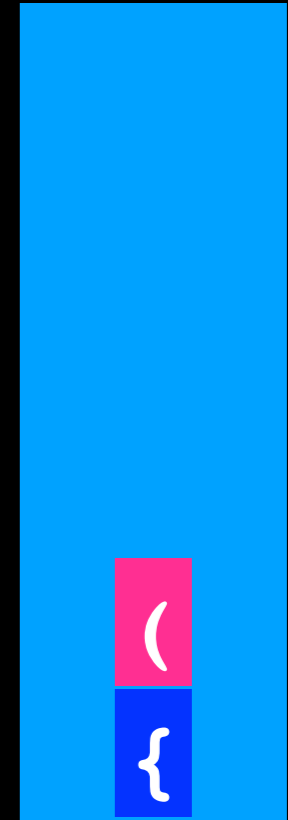
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pop

Balancing Parentheses

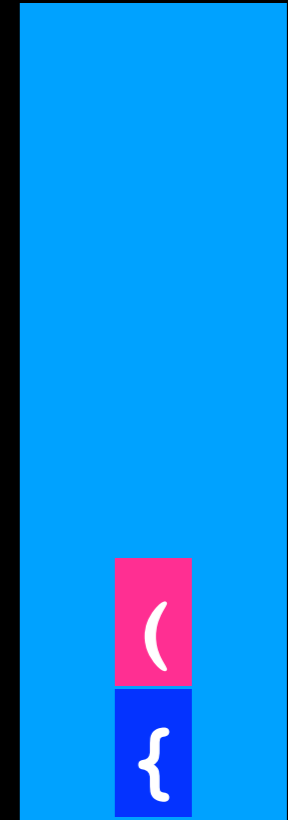
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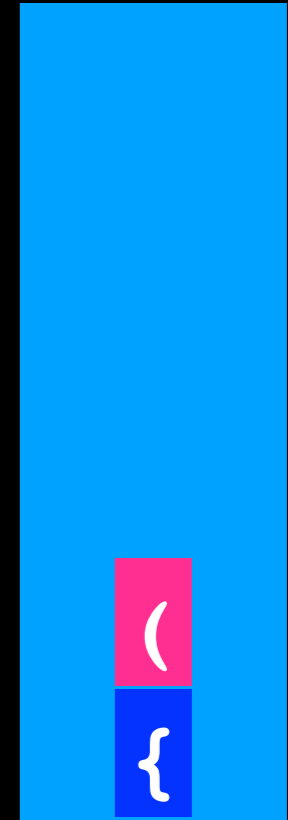
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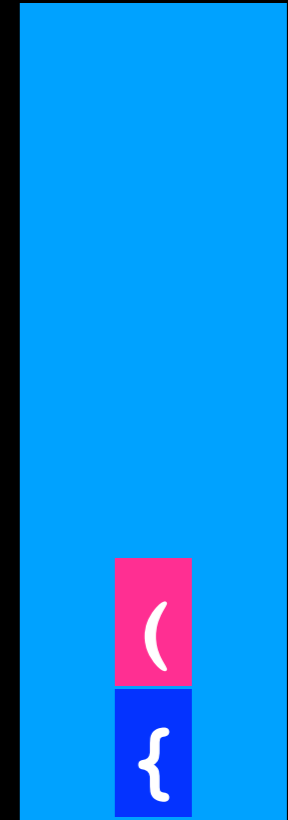
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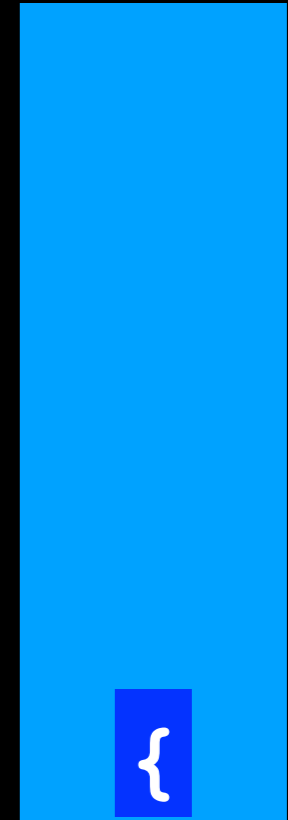
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Balancing Parentheses

```
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```

pop



Balancing Parentheses

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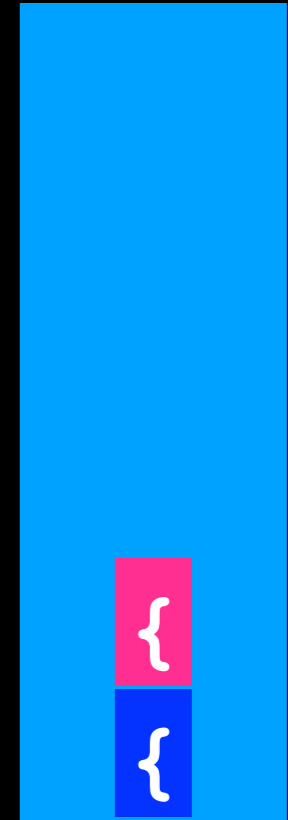


push



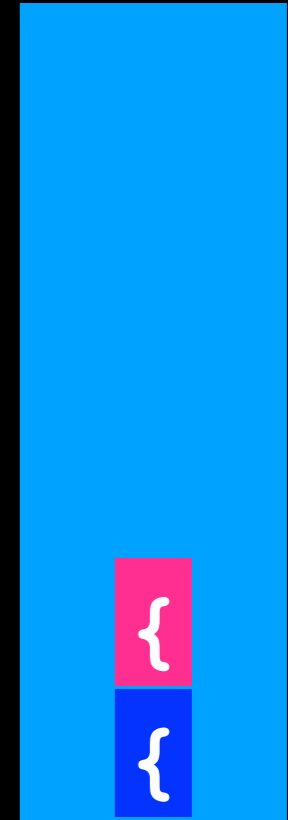
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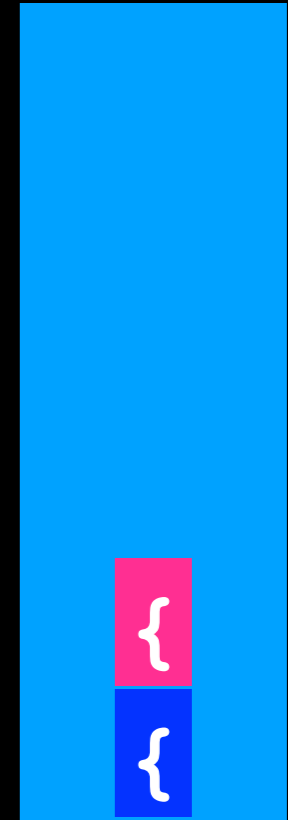
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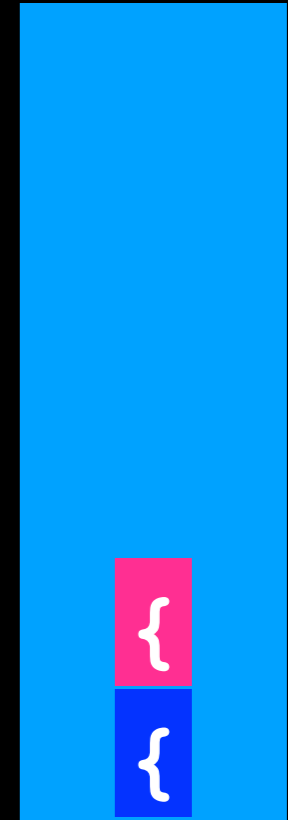
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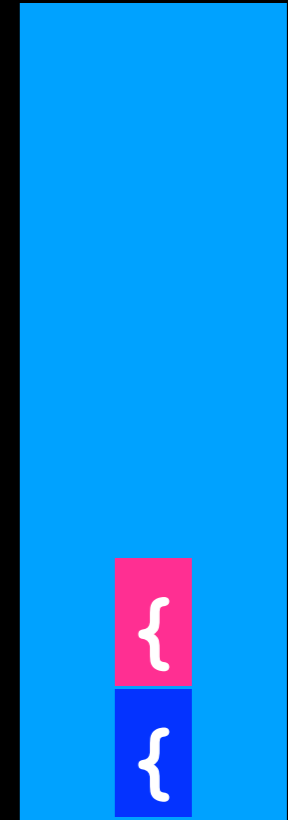
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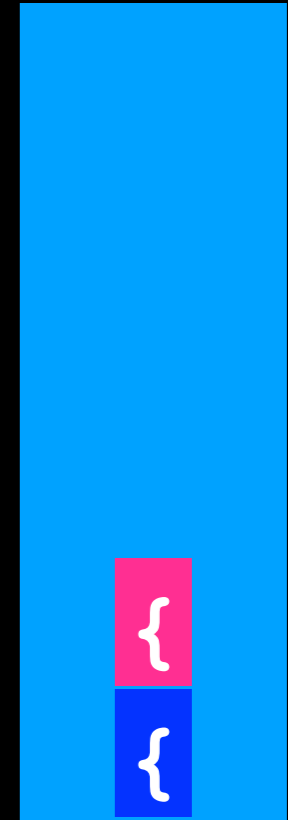
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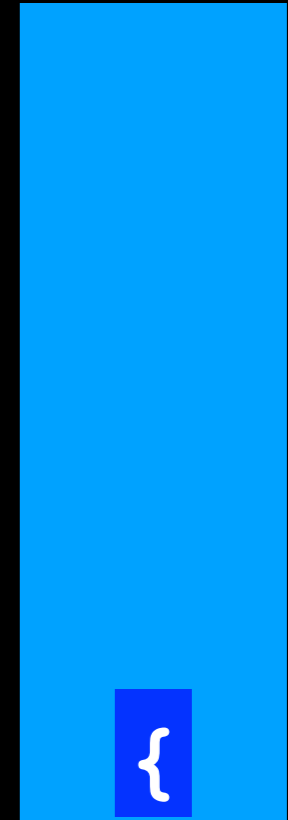
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Balancing Parentheses

```
int f() {if(x*(y+z[i])<47) {x += y}}
```

pop



Balancing Parentheses

```
int f() {if(x*(y+z[i])<47) {x += y}}
```

pop

Finished reading
Stack is empty
Parentheses are balanced



Balancing Parentheses

```
int f() {if(x*(y+z[i])<47) {x += y}
```



Finished reading
Stack not empty
Parentheses **NOT** balanced

A blue vertical bar representing a stack. At the bottom of the bar, there is a white curly brace '{' inside a small blue square, indicating the current element on the stack.

Balancing Parentheses

```
for(char ch : st)
{
    if ch is an open parenthesis character
        push it on the stack
    else if ch is a close parenthesis character
        if it matches the top of the stack
            pop the stack
        else
            return unbalanced
    // else it is not a parenthesis
}

if stack is empty
    return balanced
else
    return unbalanced
```

O(n)

Evaluating Postfix Expressions

Postfix Expressions

Operator applies to the two operands immediately preceding it

Infix:

$2 * (3 + 4)$

$2 * 3 + 4$

Postfix:

$2 3 4 + *$

$2 3 * 4 +$

Evaluating Postfix Expressions

Operator applies to the two operands immediately preceding it

Postfix:
2 3 4 + *

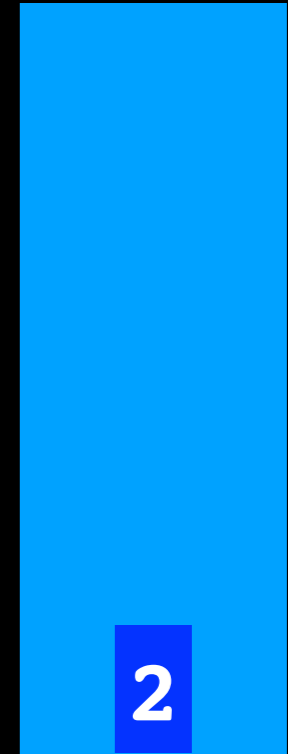
Assumptions / simplifications:

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- No unary operators
- No exponentiation operation
- Operands in string are single integer values

Evaluating Postfix Expressions

Postfix:

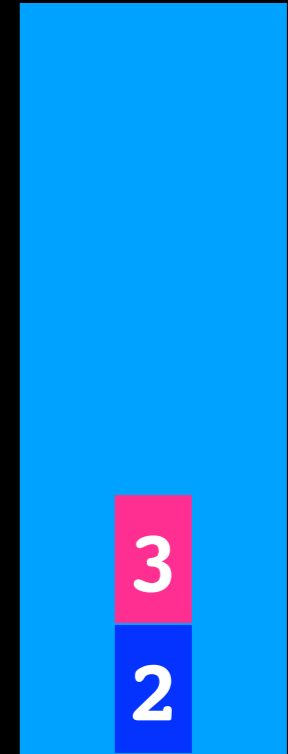
2 3 4 + *



Evaluating Postfix Expressions

Postfix:

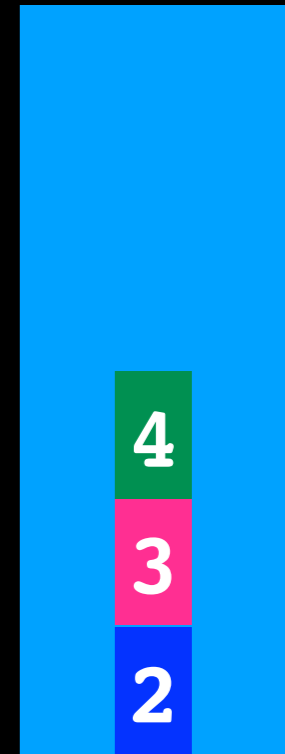
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Evaluating Postfix Expressions

Postfix:

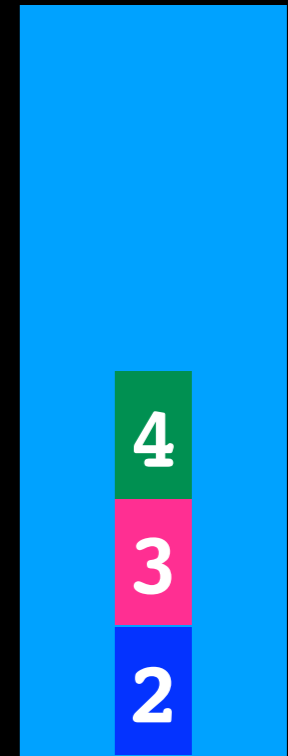

2 3 4 + *



Evaluating Postfix Expressions

Postfix:


2 3 4 + *



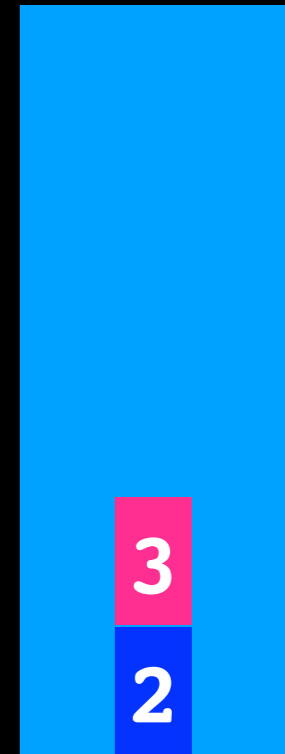
Evaluating Postfix Expressions

Postfix:

2 3 4 + *



4 +



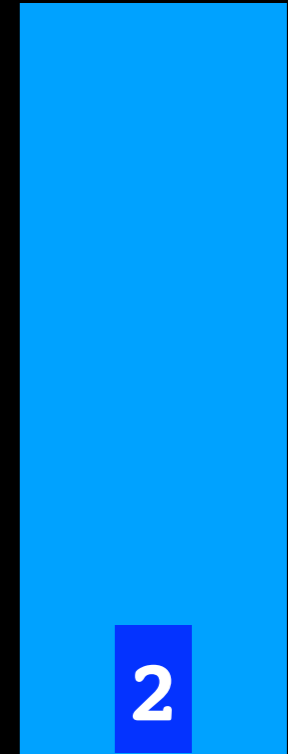
Evaluating Postfix Expressions

Postfix:

2 3 4 + *



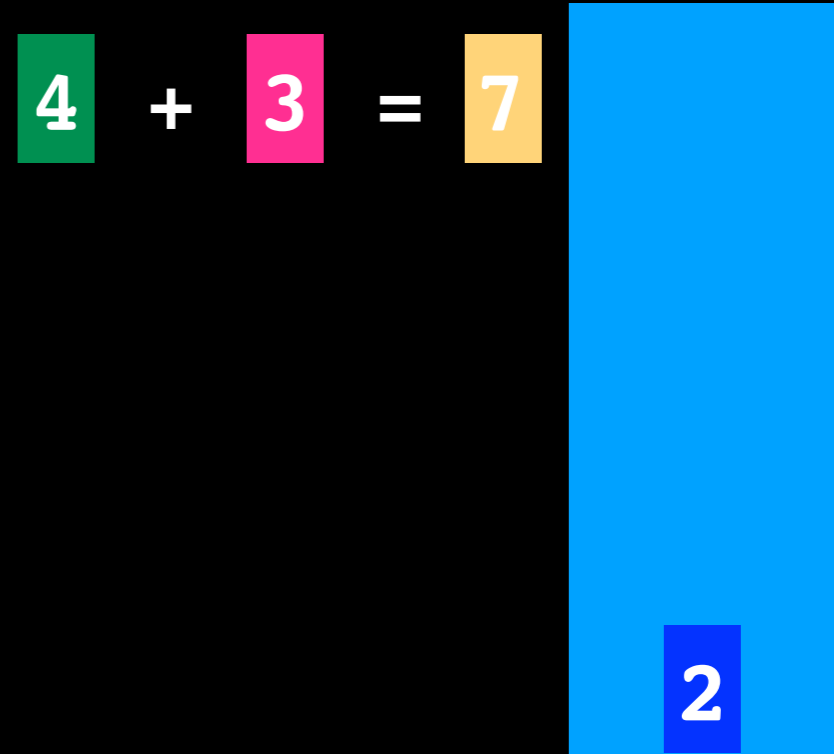
4 + 3



Evaluating Postfix Expressions

Postfix:

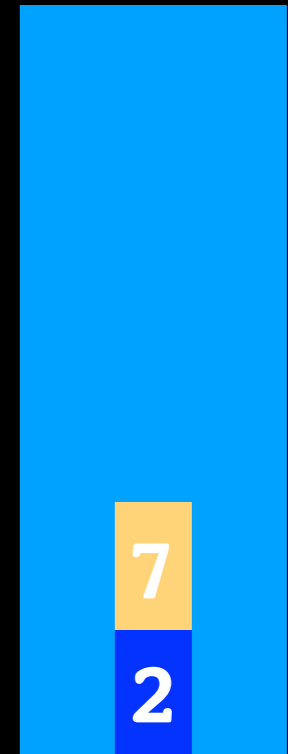

2 3 4 + *



Evaluating Postfix Expressions

Postfix:

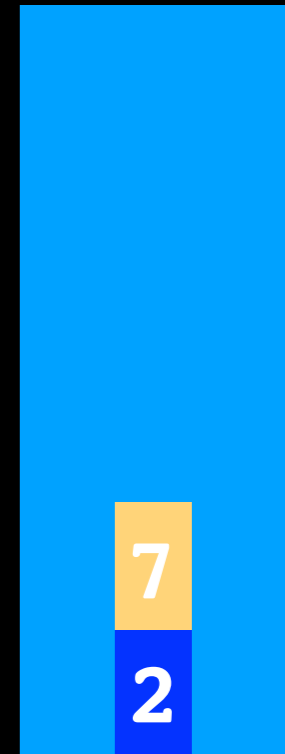
2 3 4 + *



Evaluating Postfix Expressions

Postfix:

2 3 4 + *



Evaluating Postfix Expressions

Postfix:

2 3 4 + *
↑

7



Evaluating Postfix Expressions

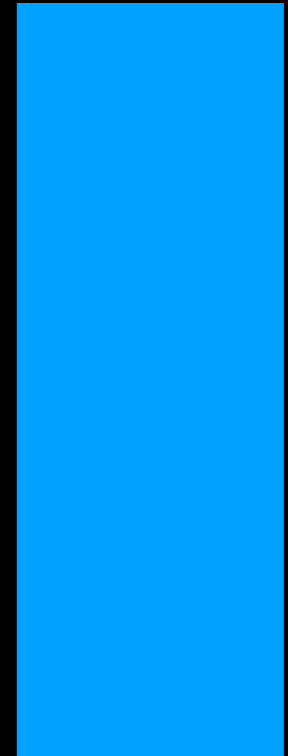
Postfix:

2 3 4 + *
↑

7

*

2



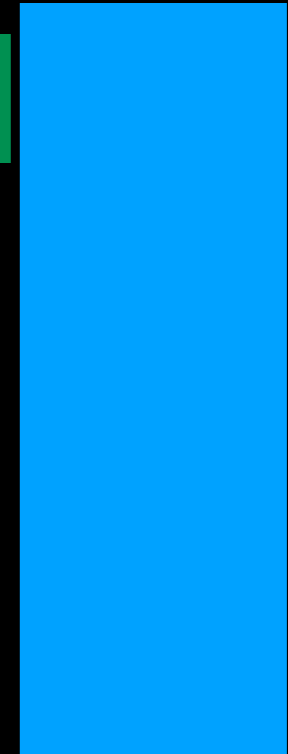
Evaluating Postfix Expressions

Postfix:

2 3 4 + *



7 * 2 = 14



Evaluating Postfix Expressions

Postfix:

2 3 4 + *



Done reading string
The top of the stack
is the result

14



Evaluating Postfix Expressions

Operator applies to the two operands immediately preceding it

Postfix:

2 3 * 4 +

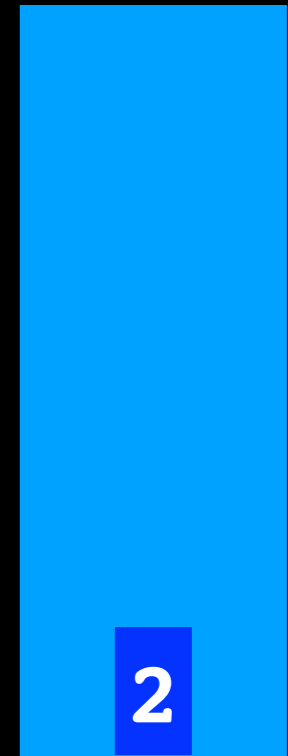
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Evaluating Postfix Expressions

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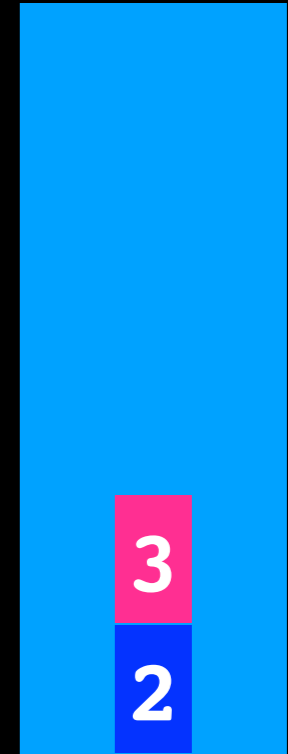
2 3 * 4 +



Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



Evaluating Postfix Expressions

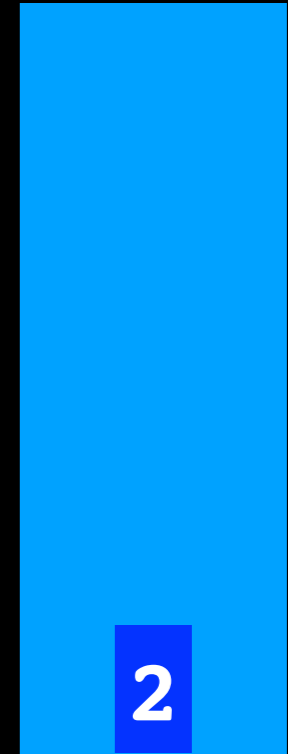
Postfix:

2 3 * 4 +



3

*

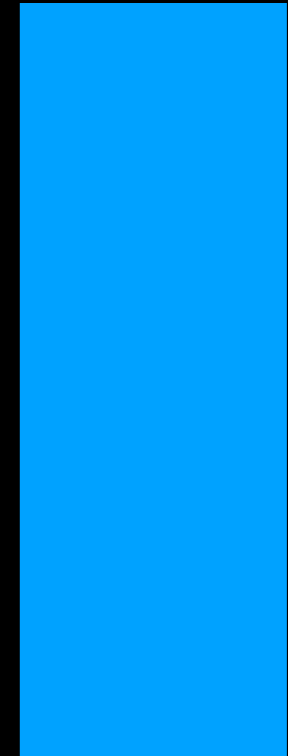


Evaluating Postfix Expressions

Postfix:

2 3 * 4 +
↑

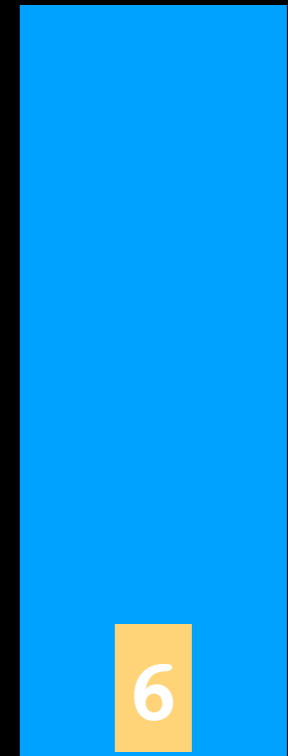
3 * 2 = 6



Evaluating Postfix Expressions

Postfix:

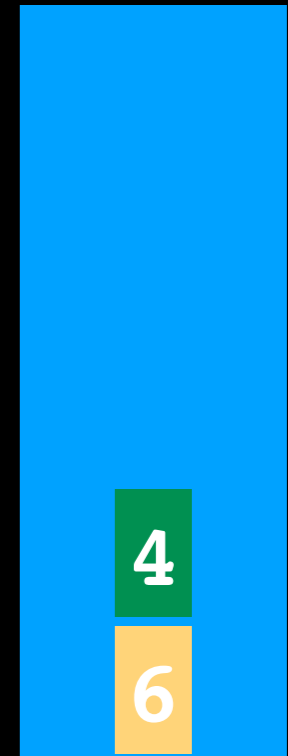
2 3 * 4 +



Evaluating Postfix Expressions

Postfix:

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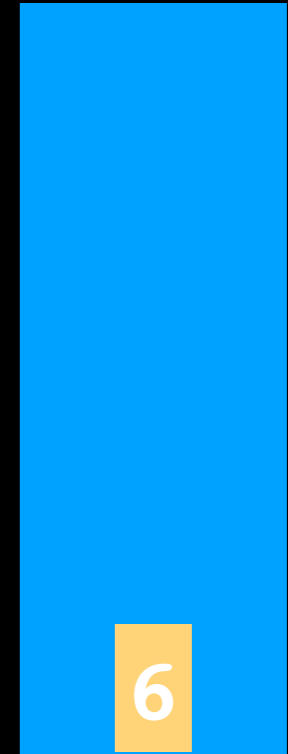


Evaluating Postfix Expressions

Postfix:

2 3 * 4 +
↑

4 +



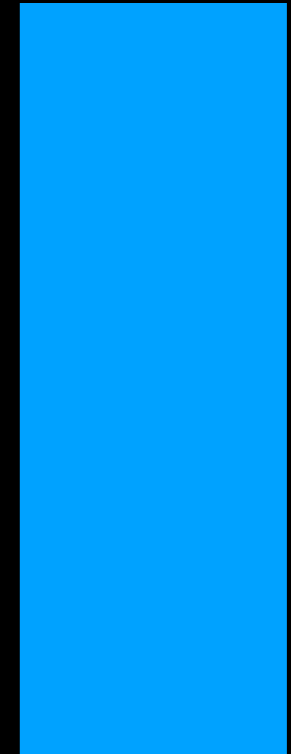
Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



$$4 + 6 = 10$$



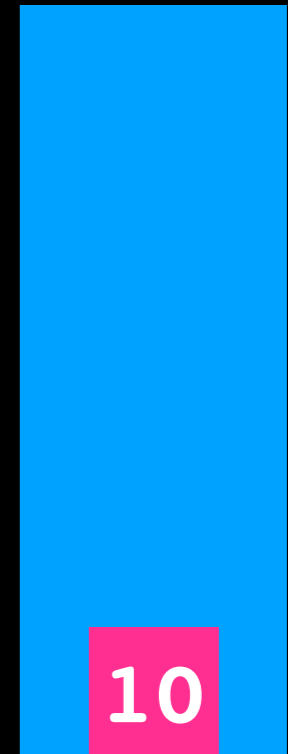
Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



Done reading string
The top of the stack
is the result



Evaluating Postfix Expressions

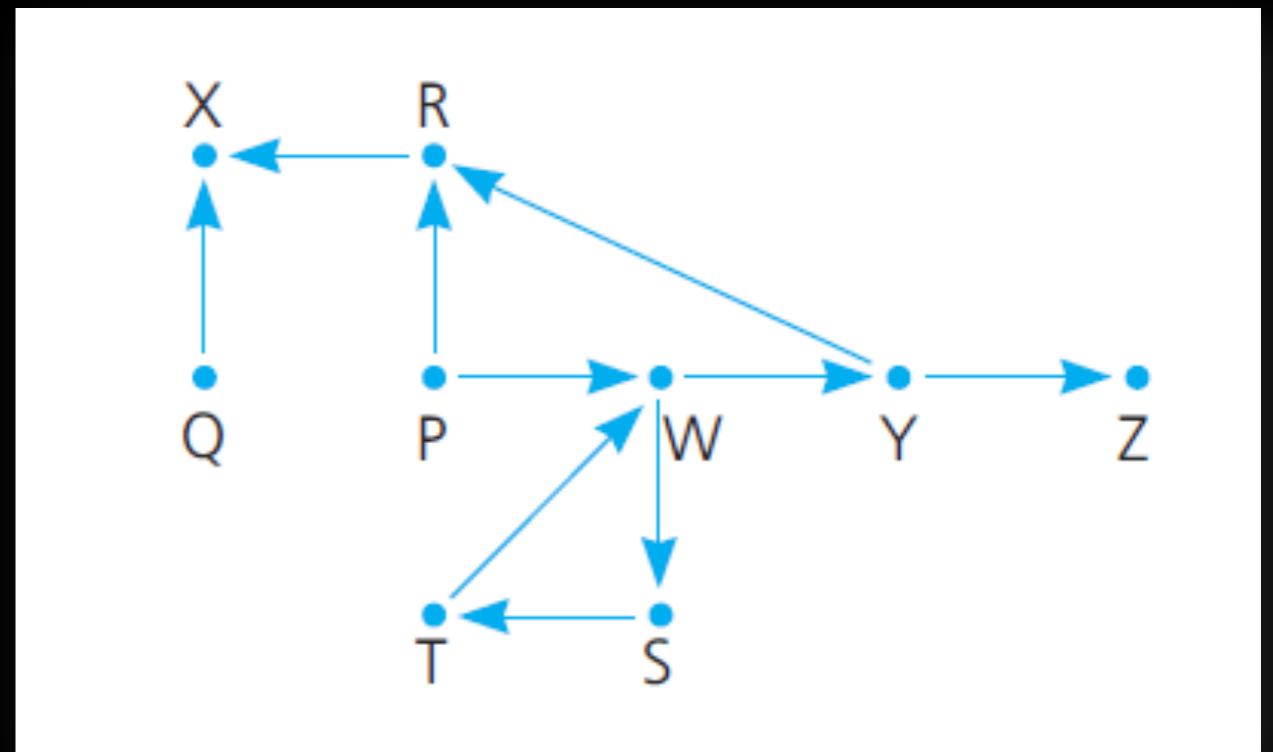
```
for(char ch : st)
{
    if ch is an operand
        push it on the stack
    else // ch is an operator op
    {
        //evaluate and push the result
        operand2 = pop stack
        operand1 = pop stack
        result = operand1 op operand2
        push result on stack
    }
}
```

O(n)

Search a Flight Map

Fly from Origin to Destination following map

1. Reach destination
2. Reach city with no departing flights (dead end)
3. Go in circles forever



Backtracking

Avoid dead end by backtracking

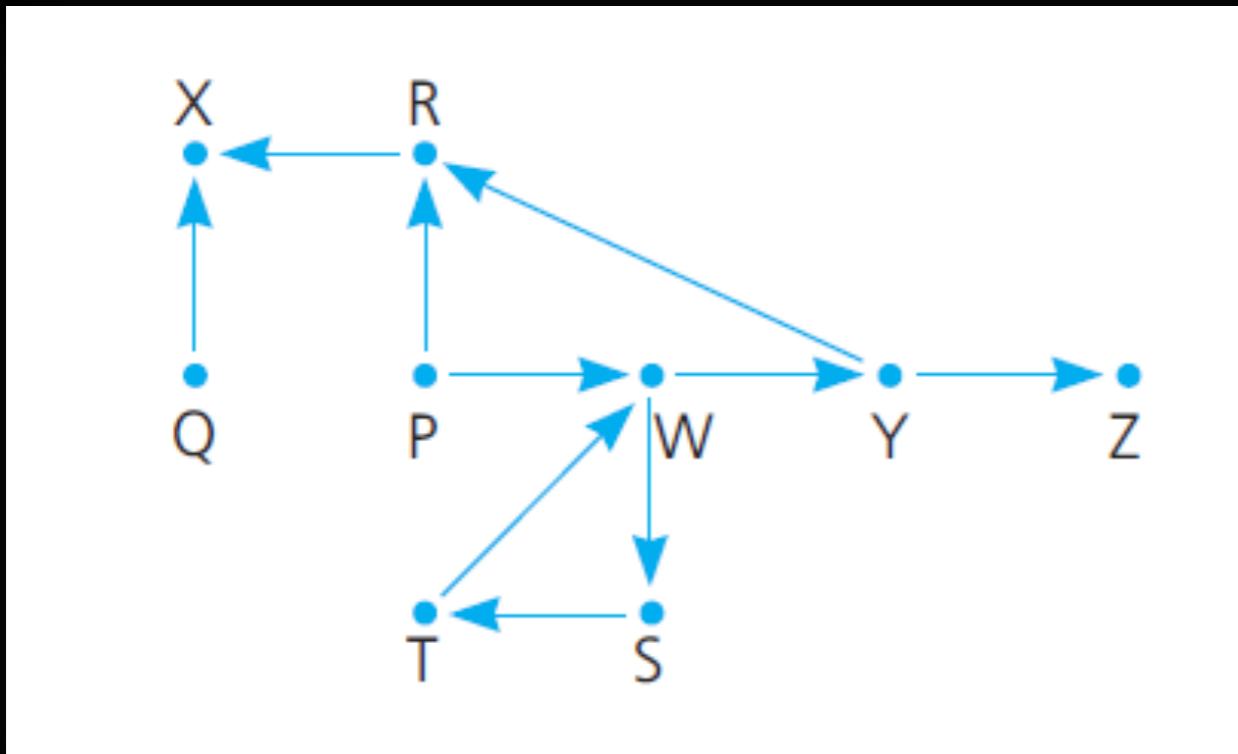
C = visited

C = backtracked

Avoid traveling in circles by marking visited cities

Origin = P , **Destination = Z**

P



Backtracking

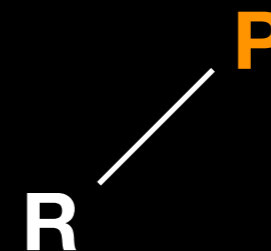
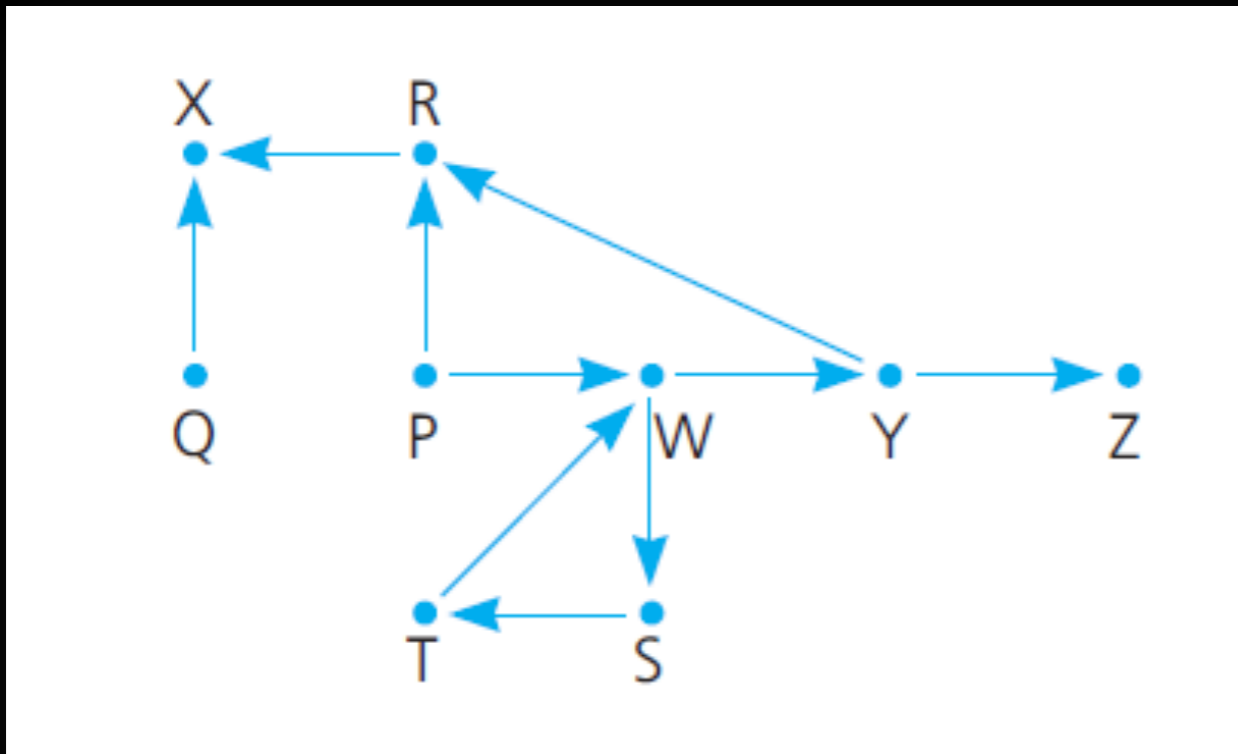
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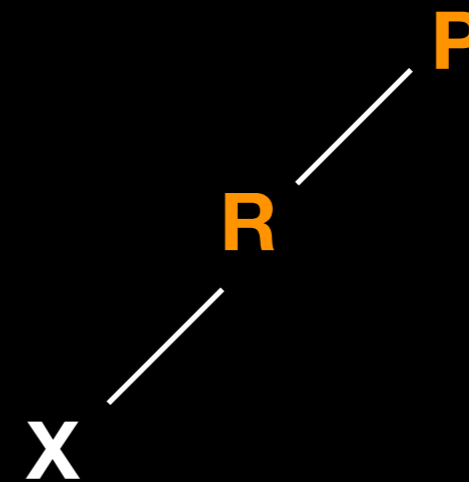
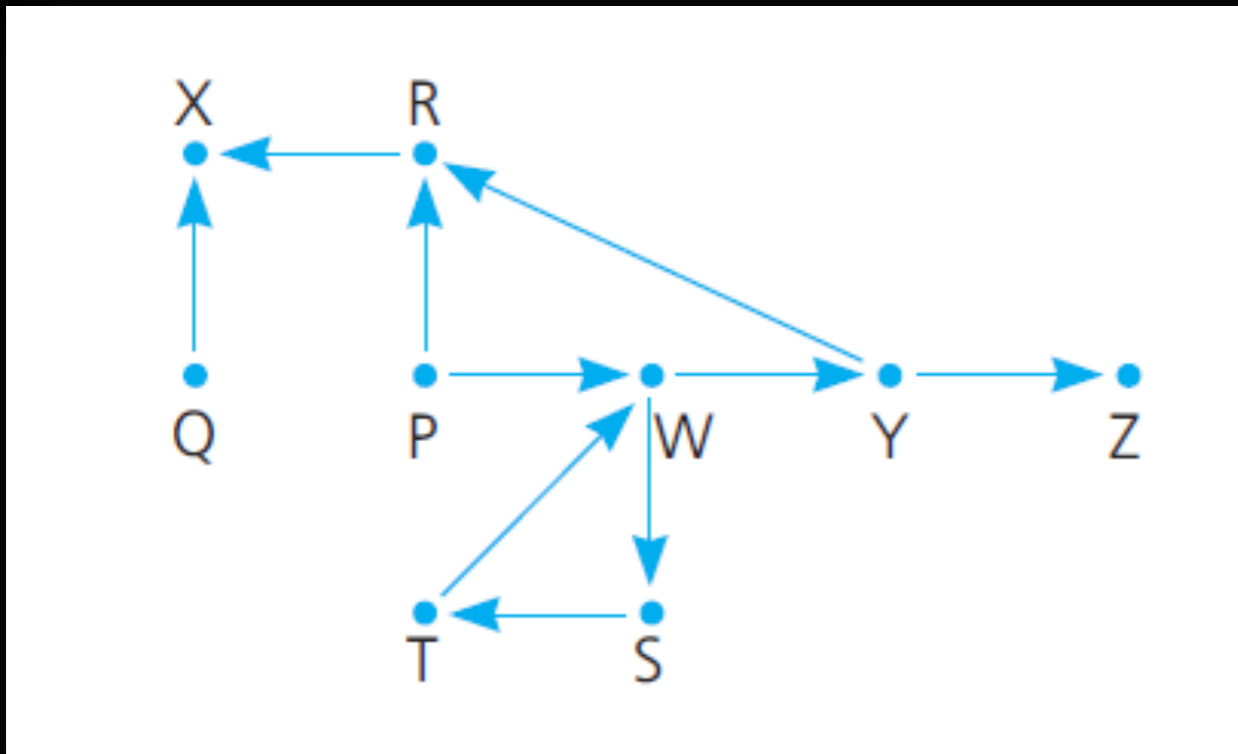
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Backtracking

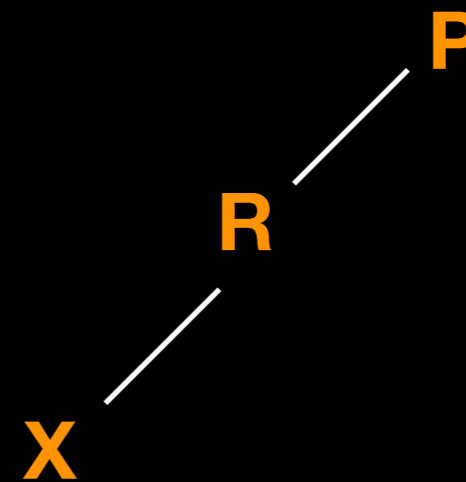
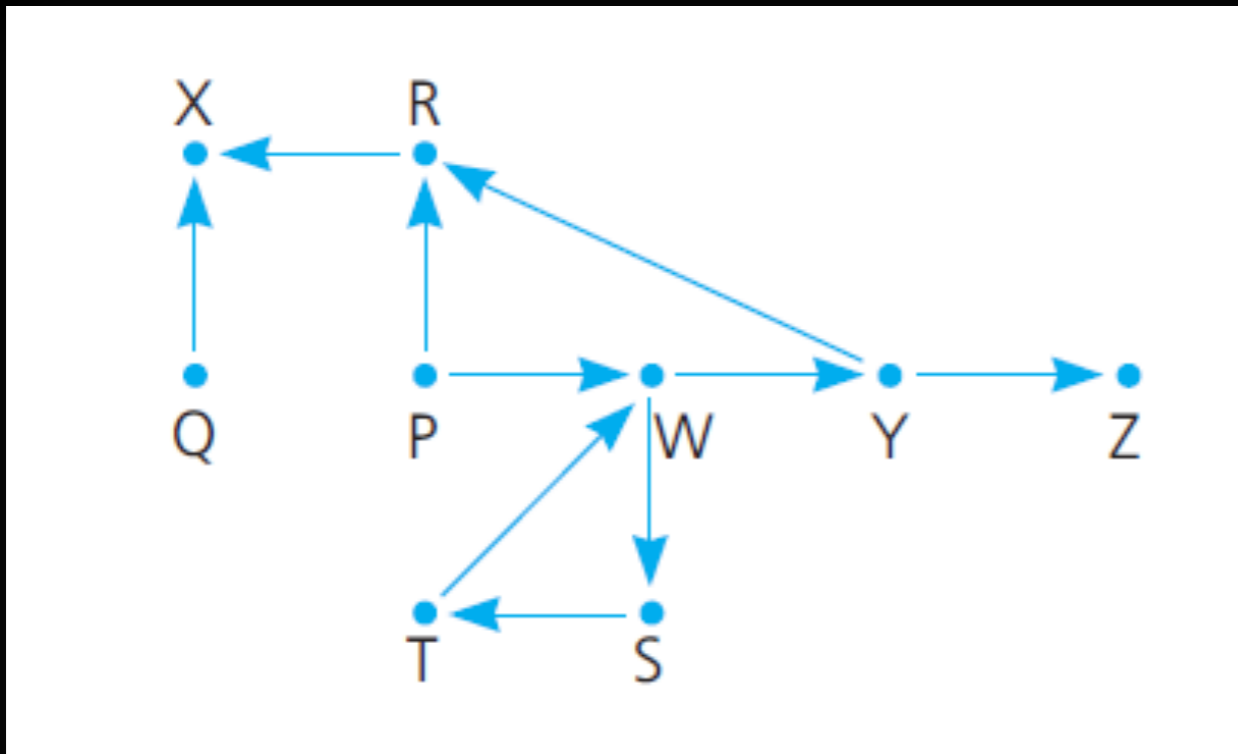
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Backtracking

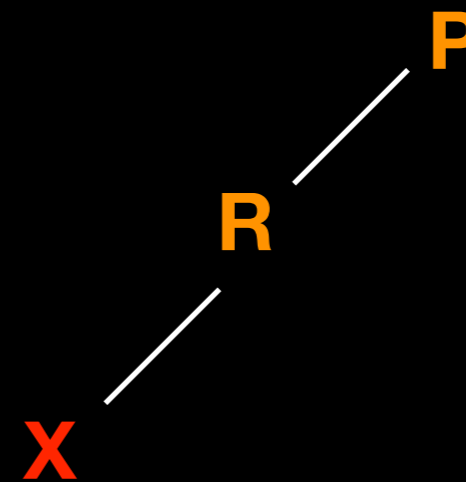
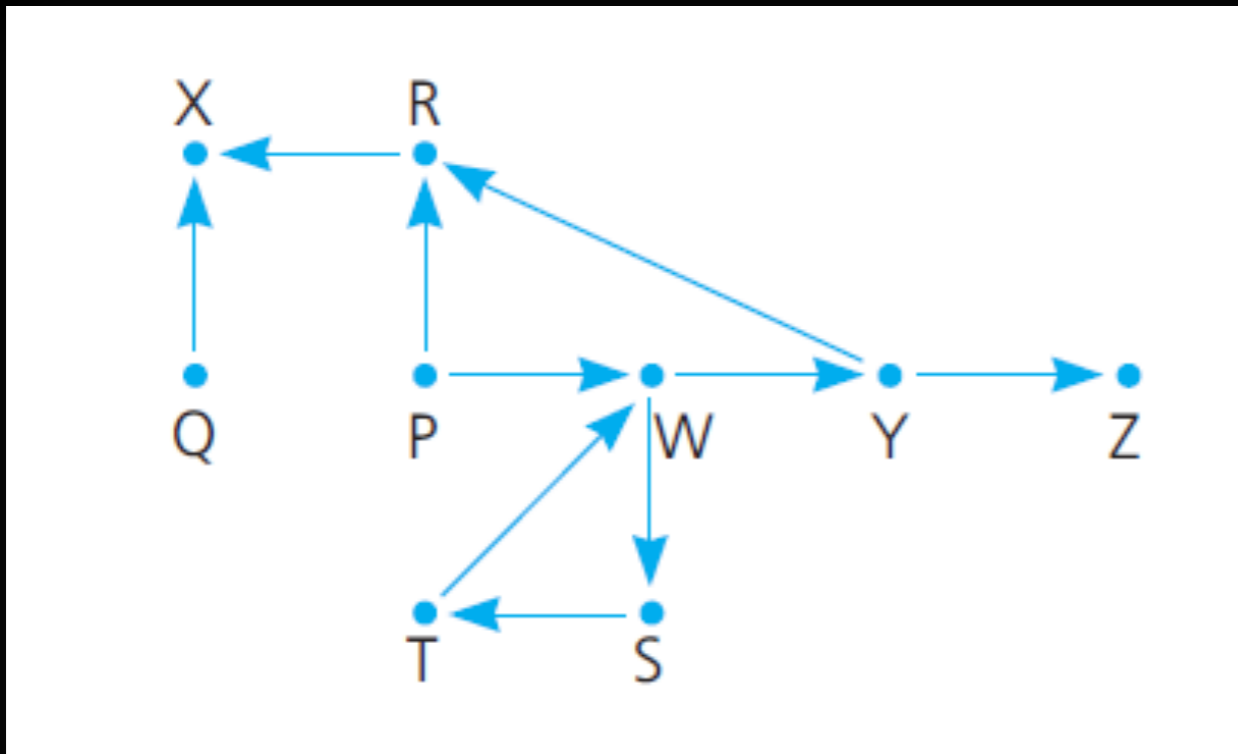
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Backtracking

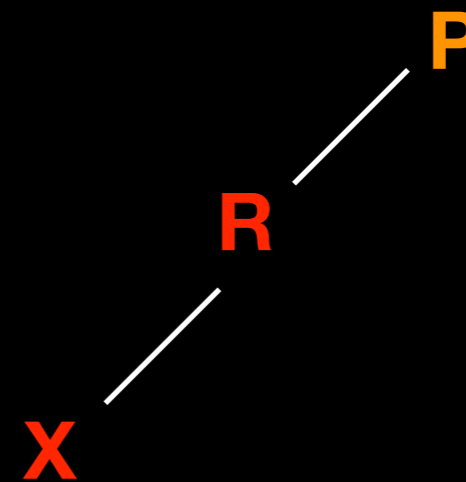
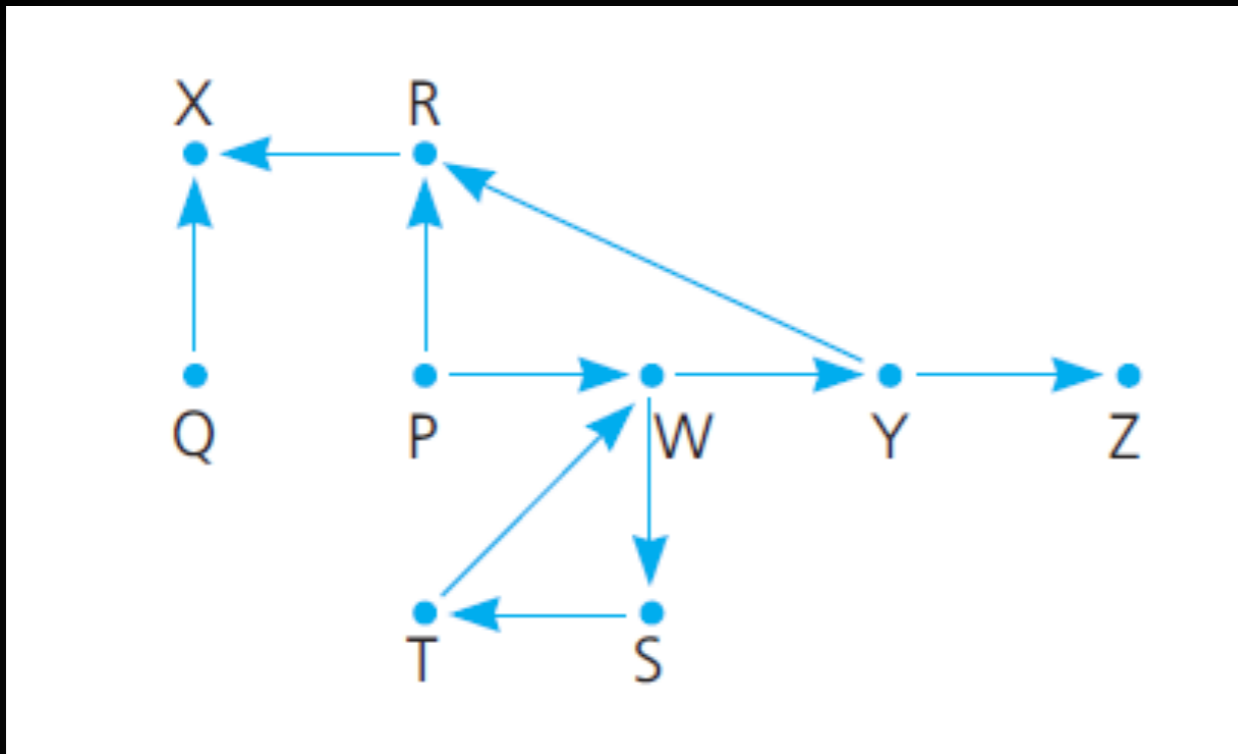
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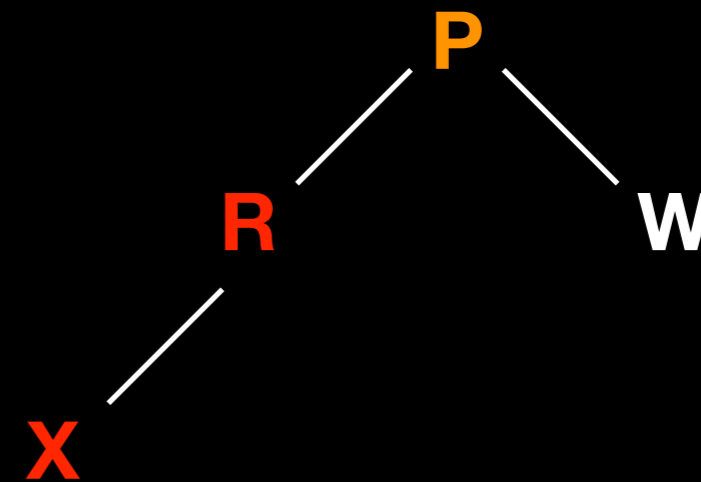
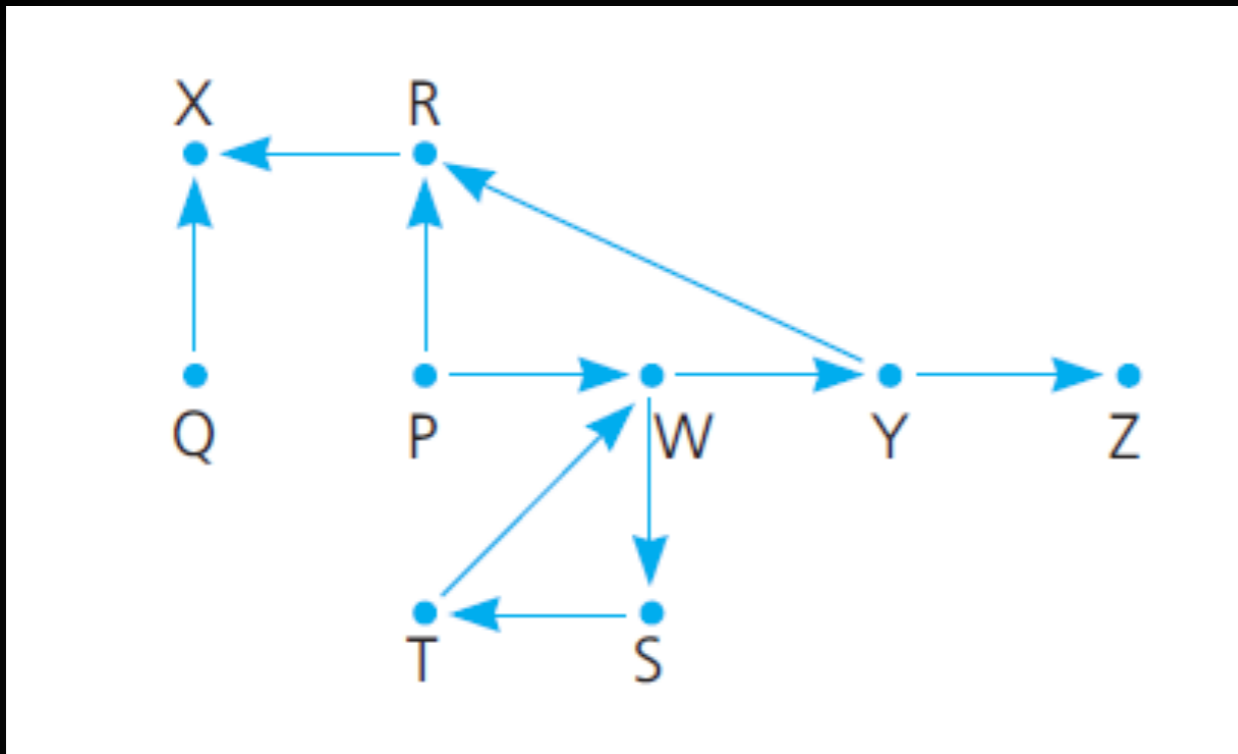
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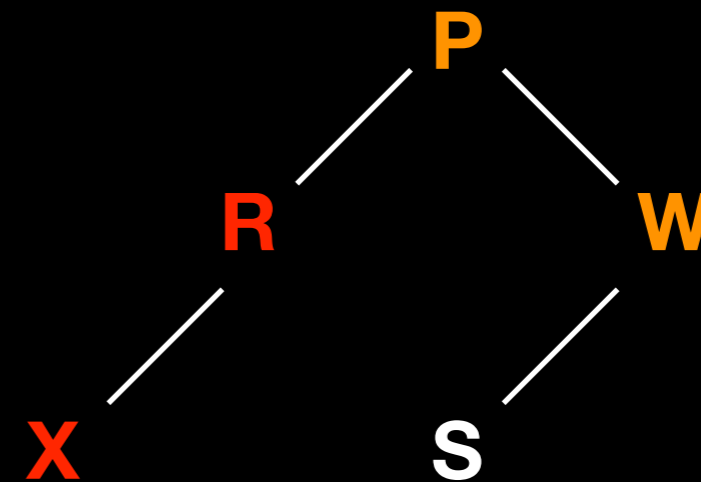
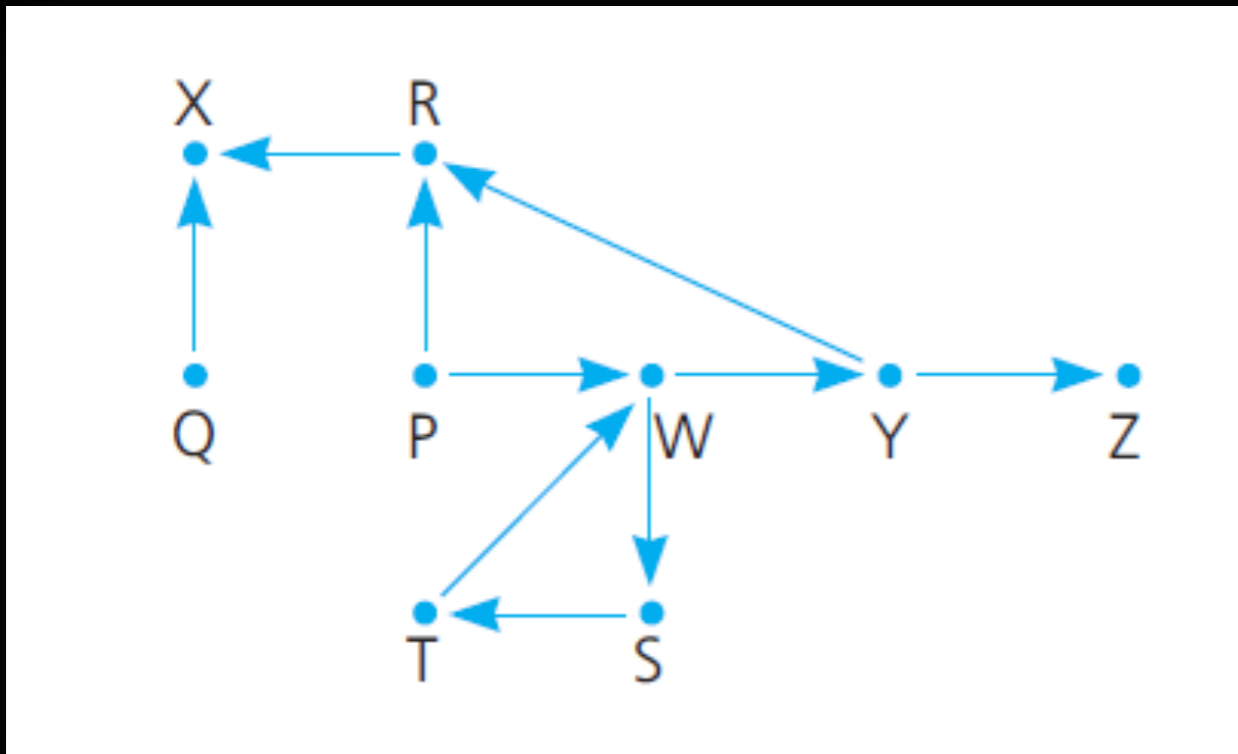
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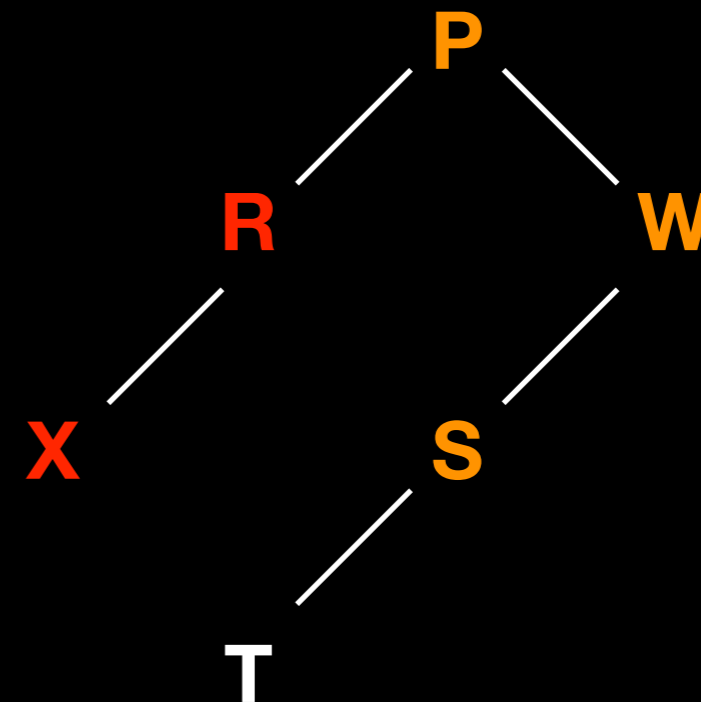
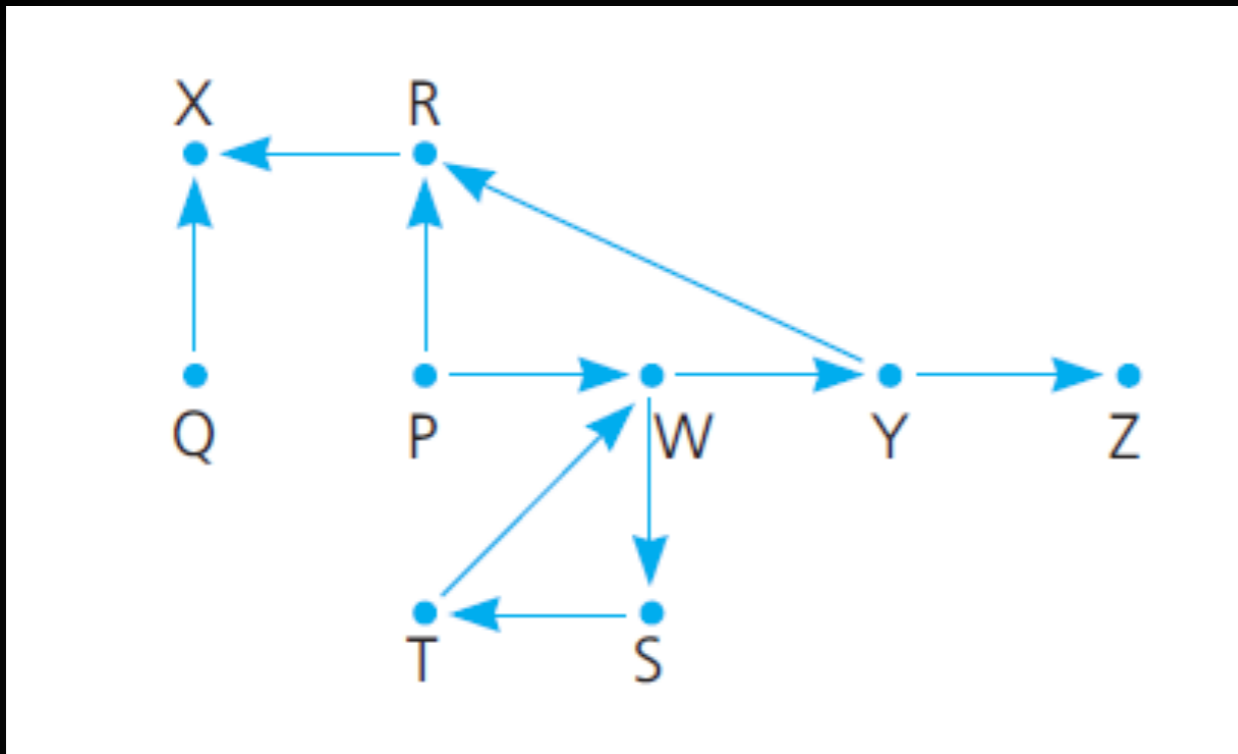
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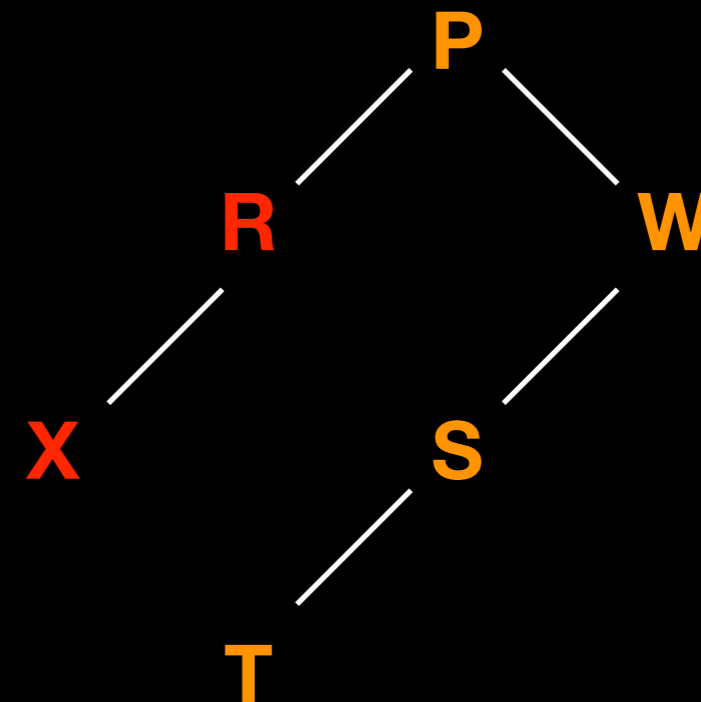
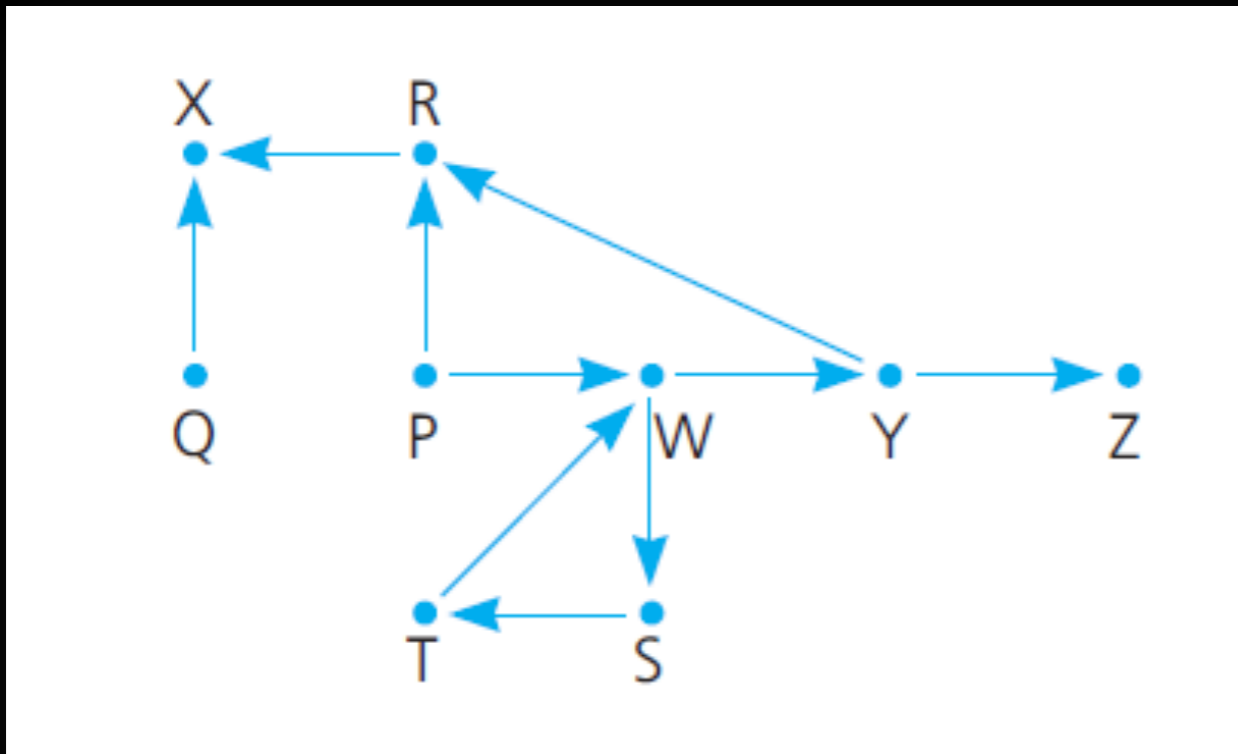
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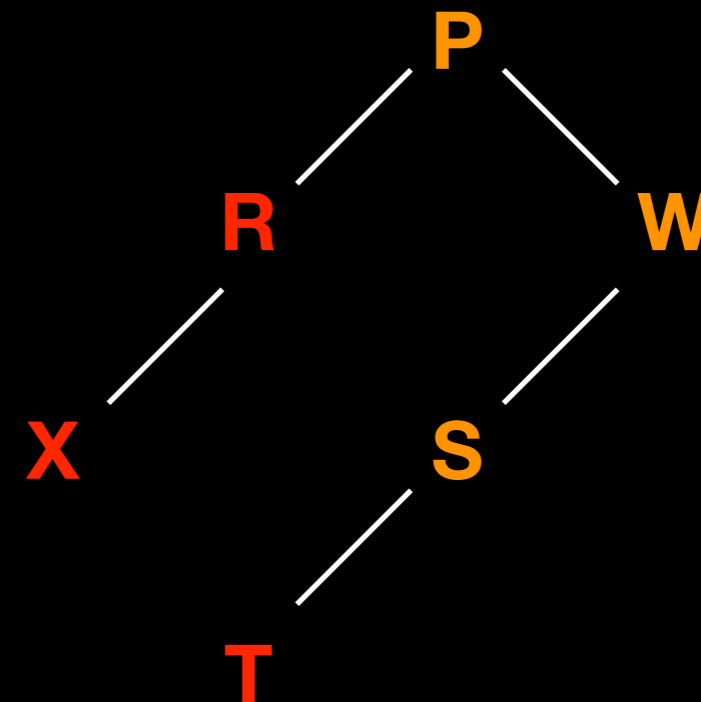
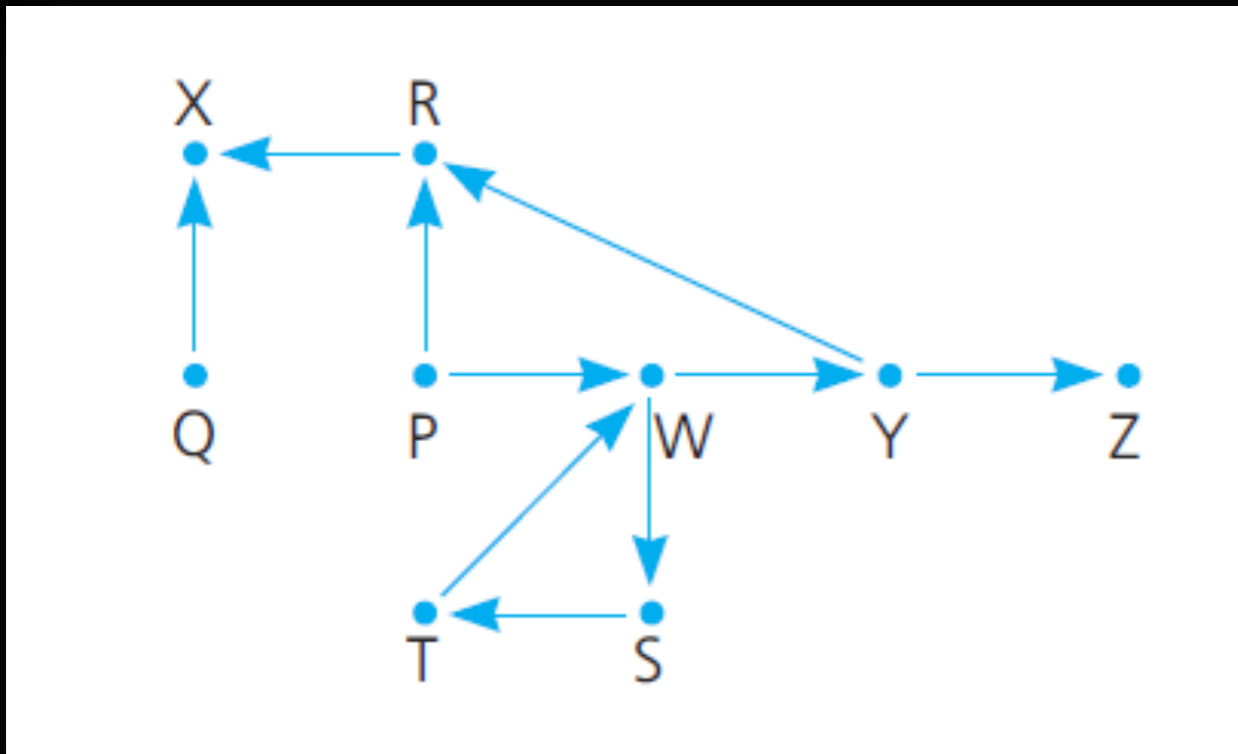
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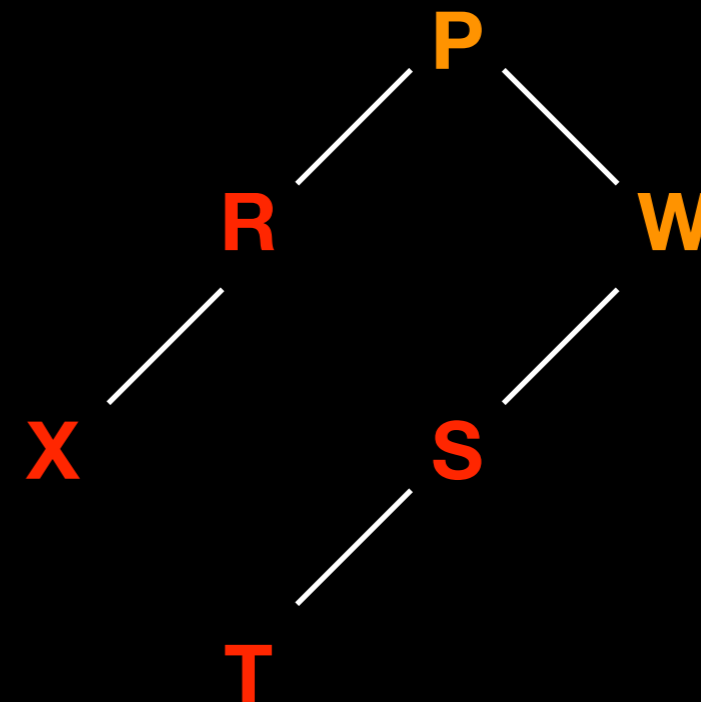
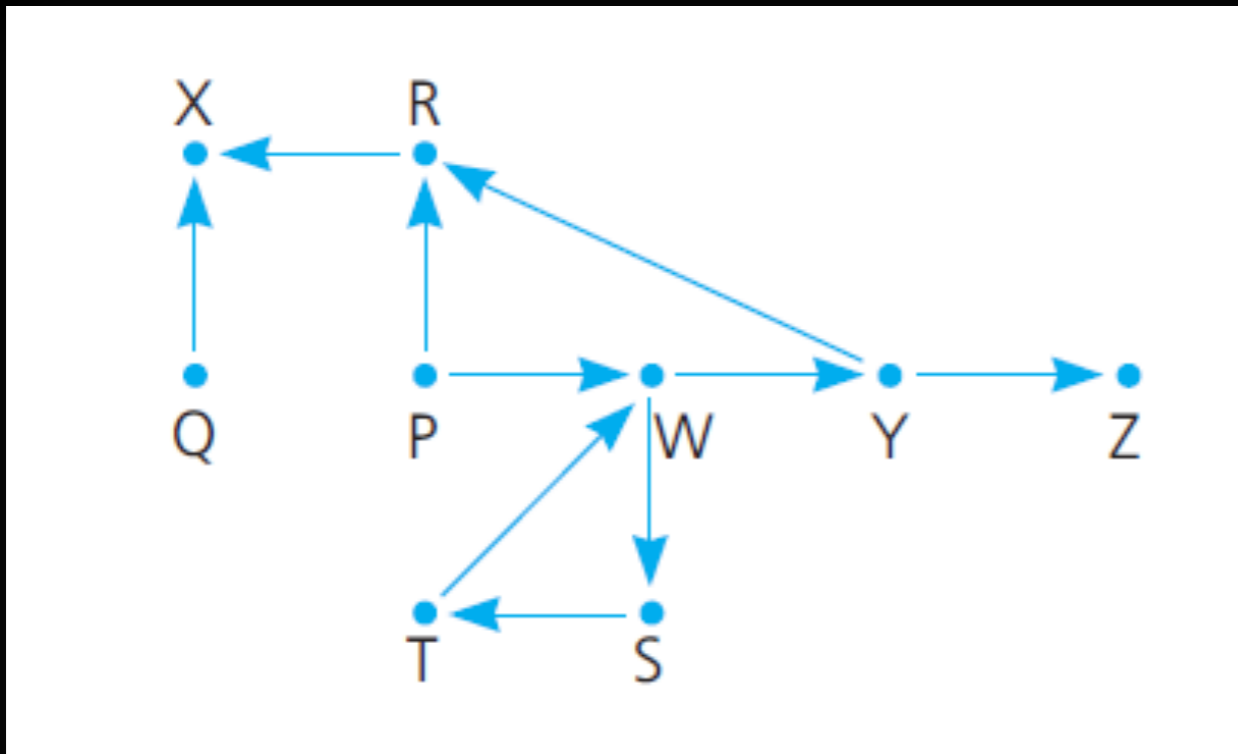
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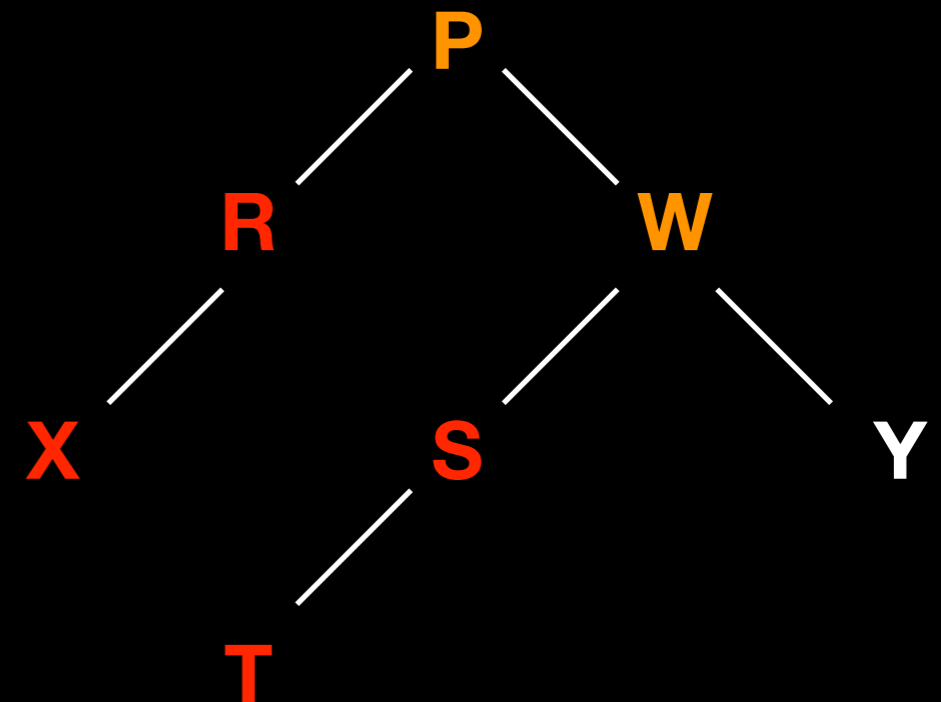
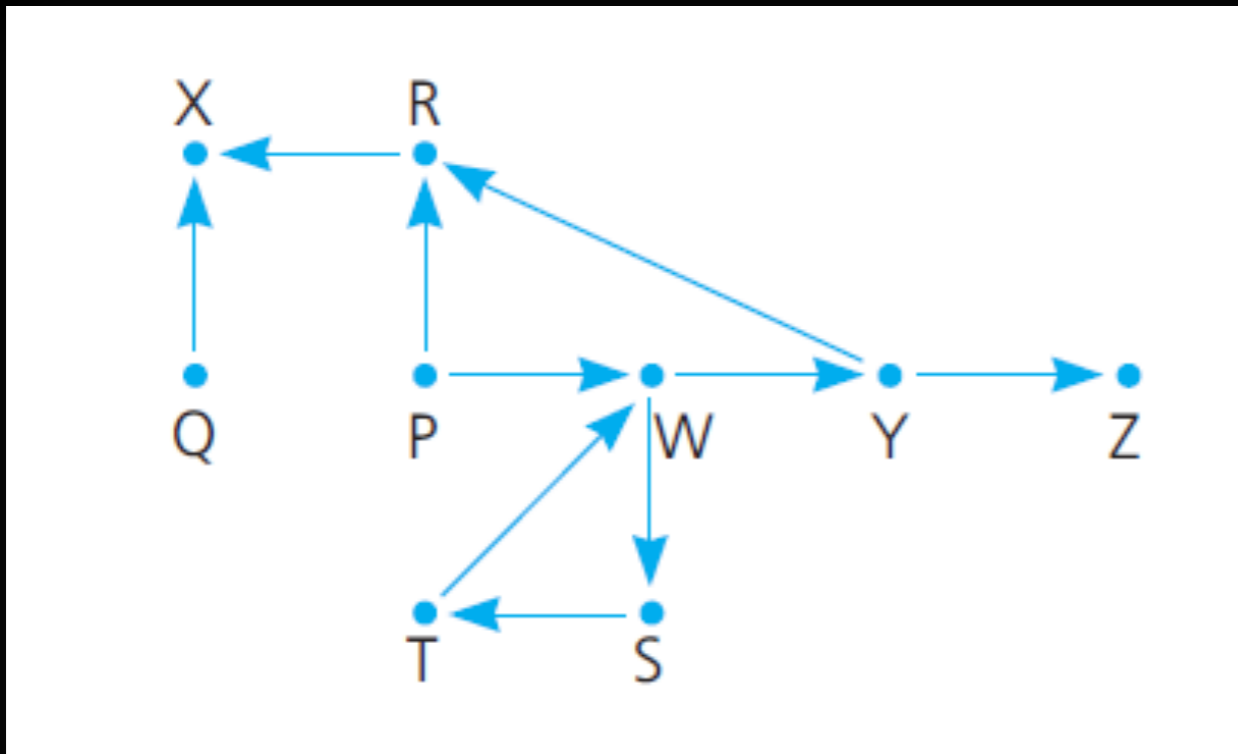
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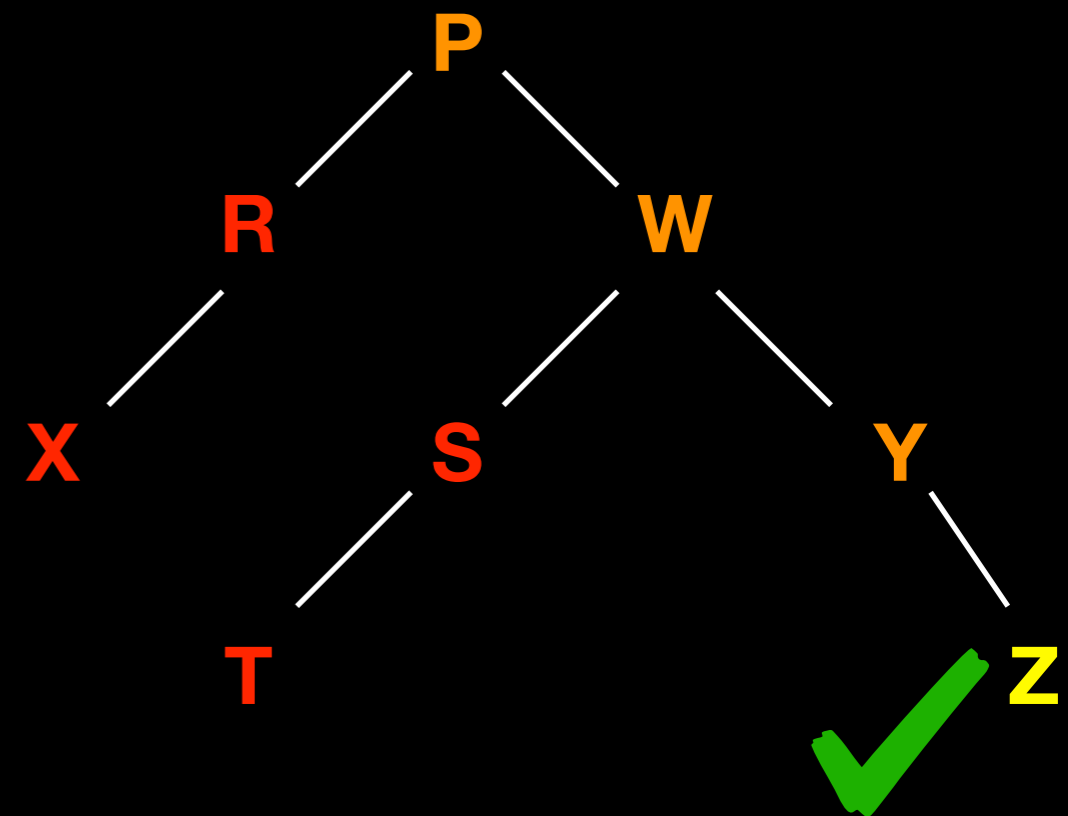
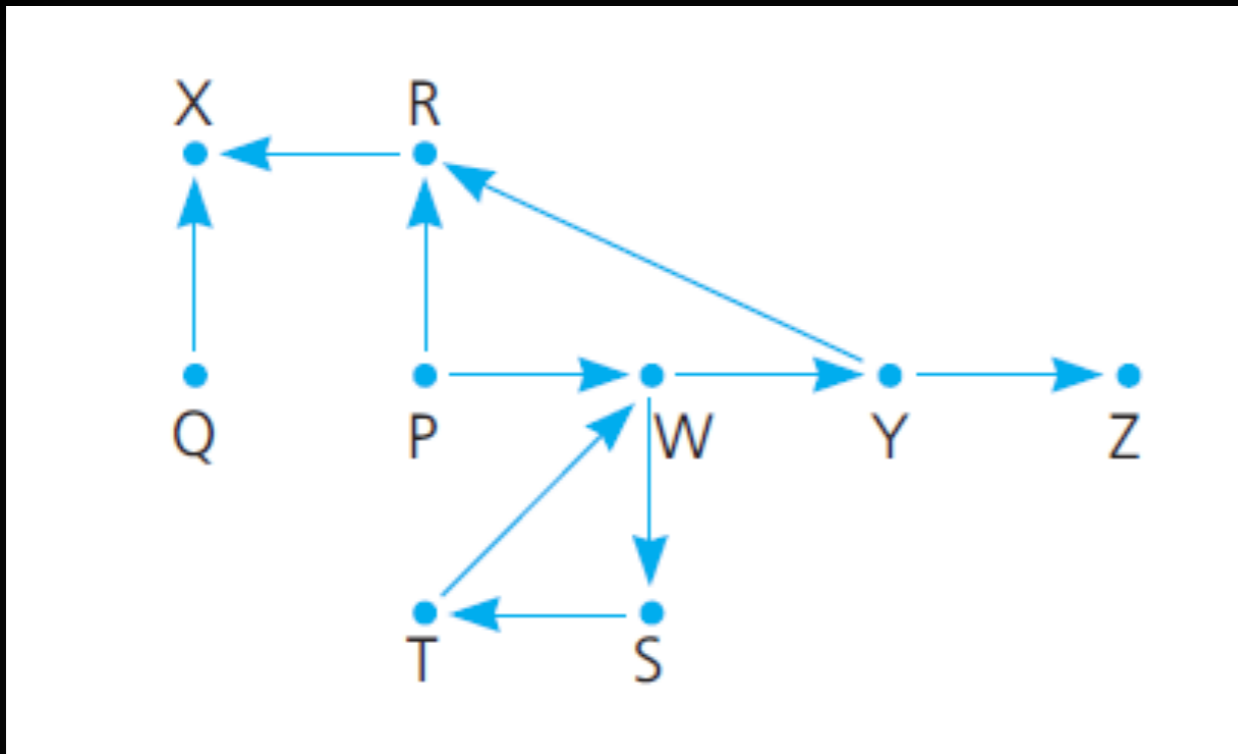
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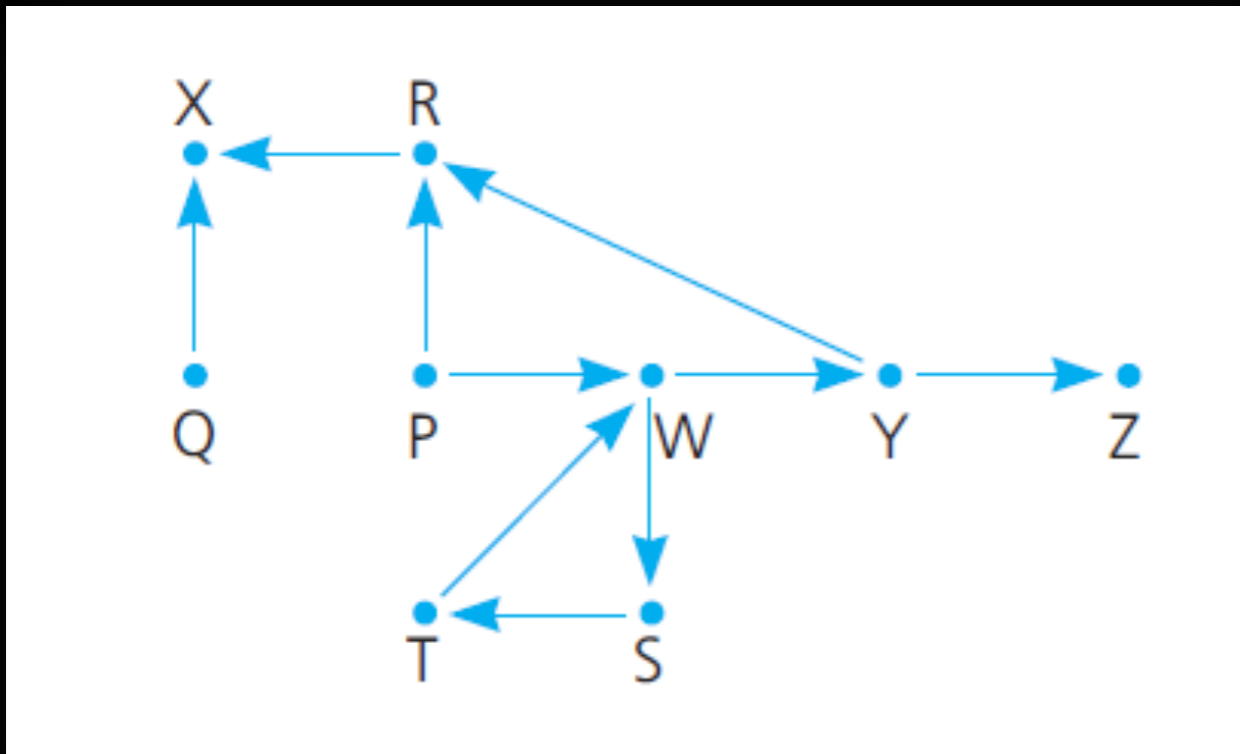
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Origin = P , **Destination = Z**



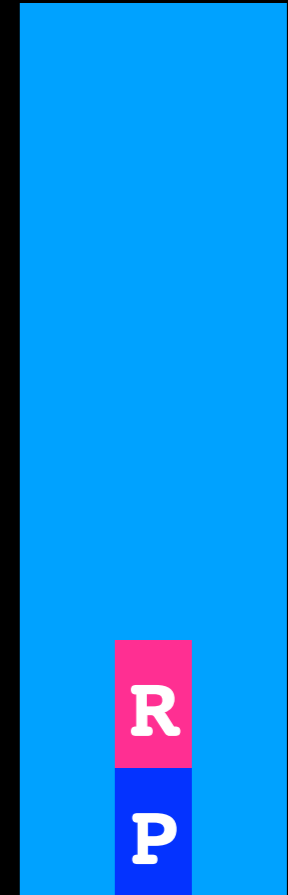
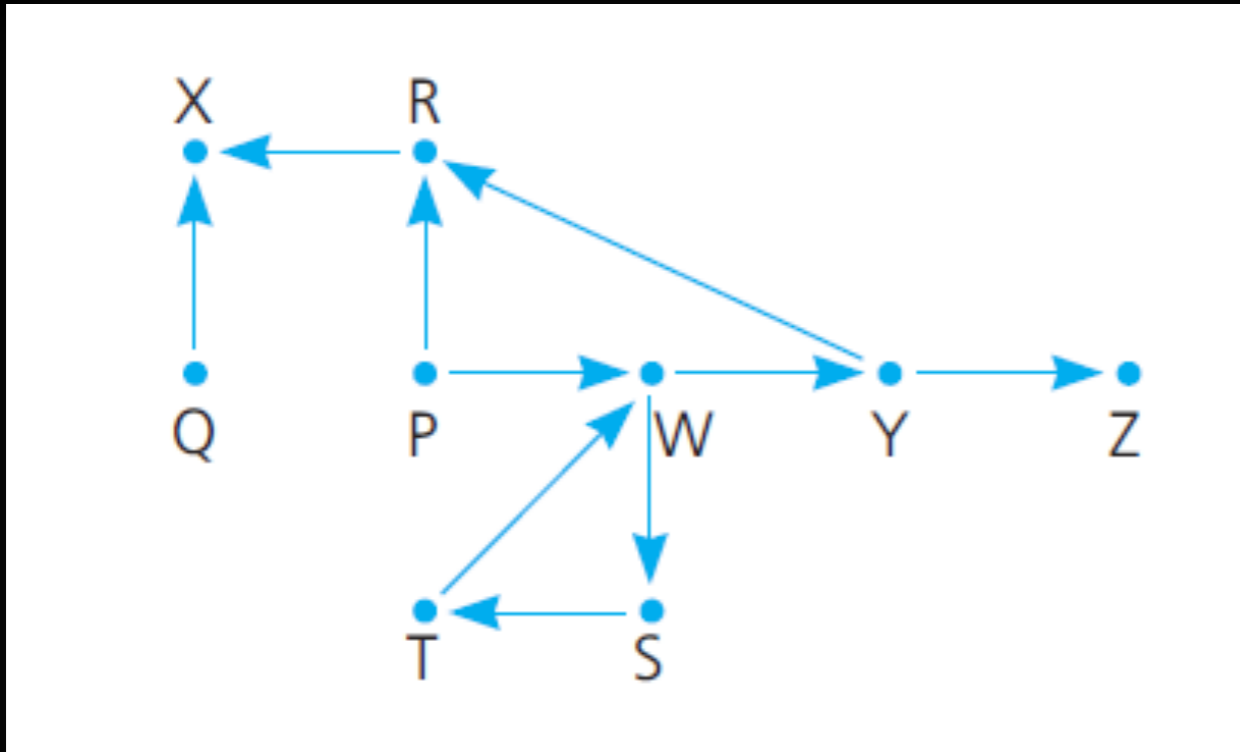
Backtracking

Origin = P , Destination = Z



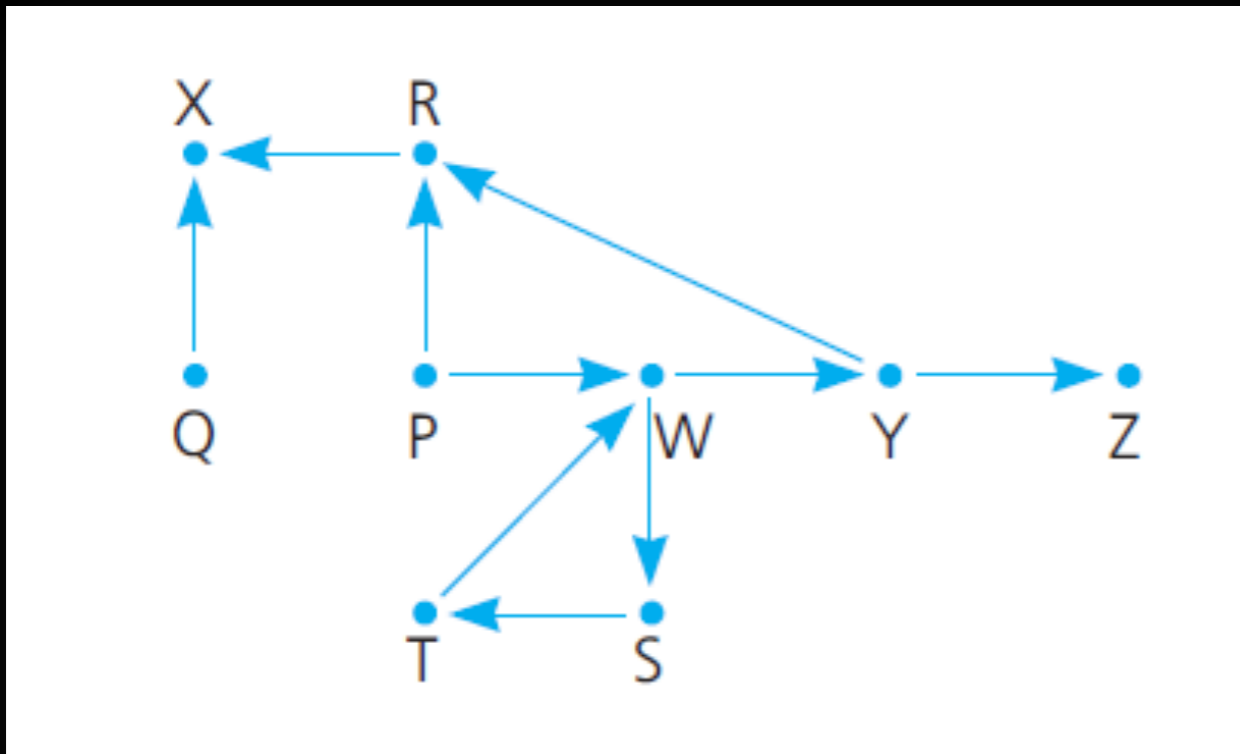
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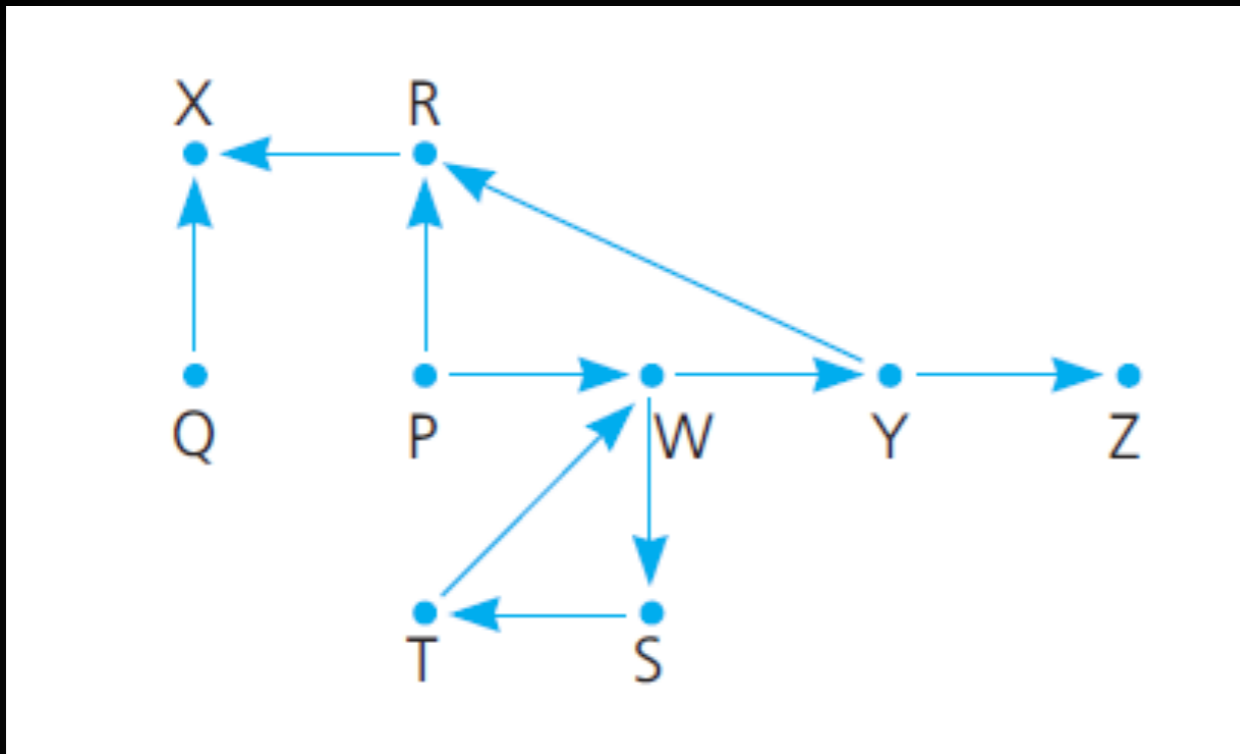
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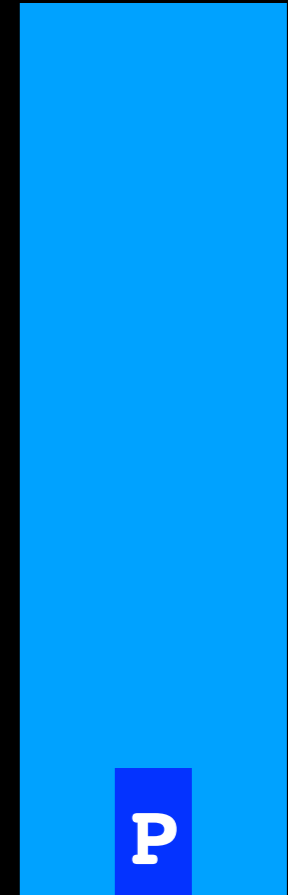
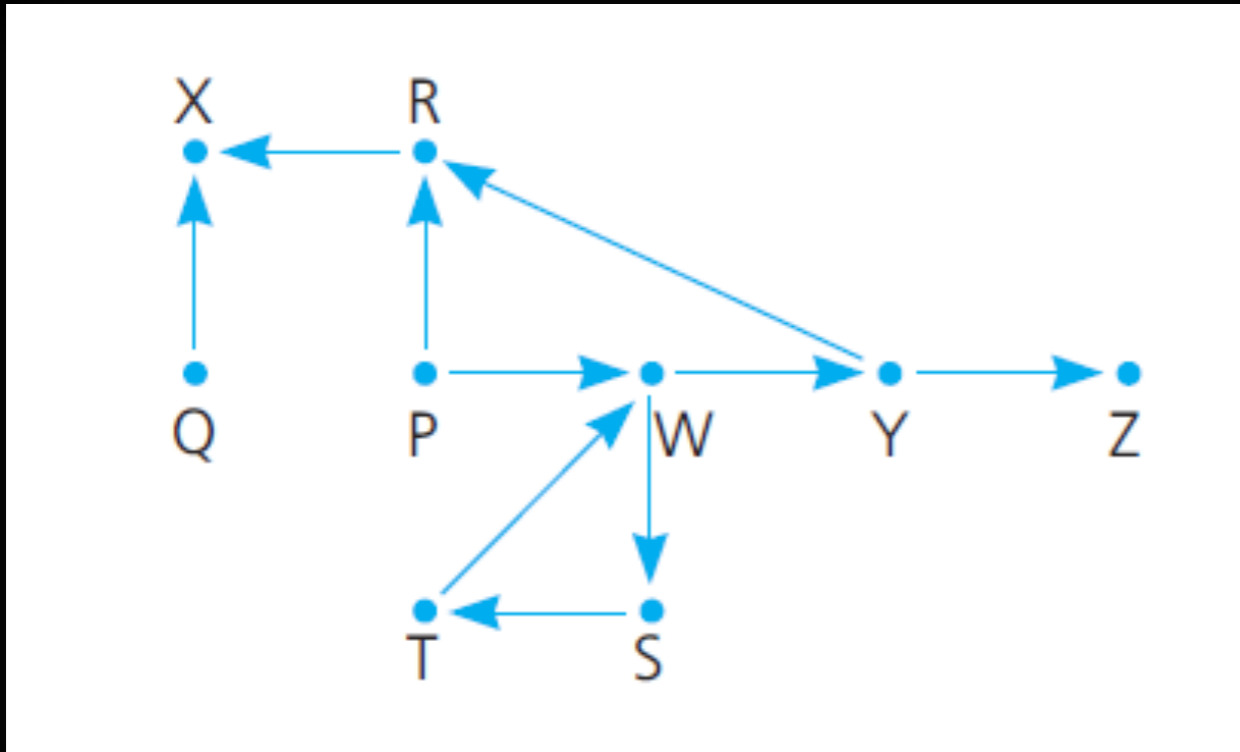
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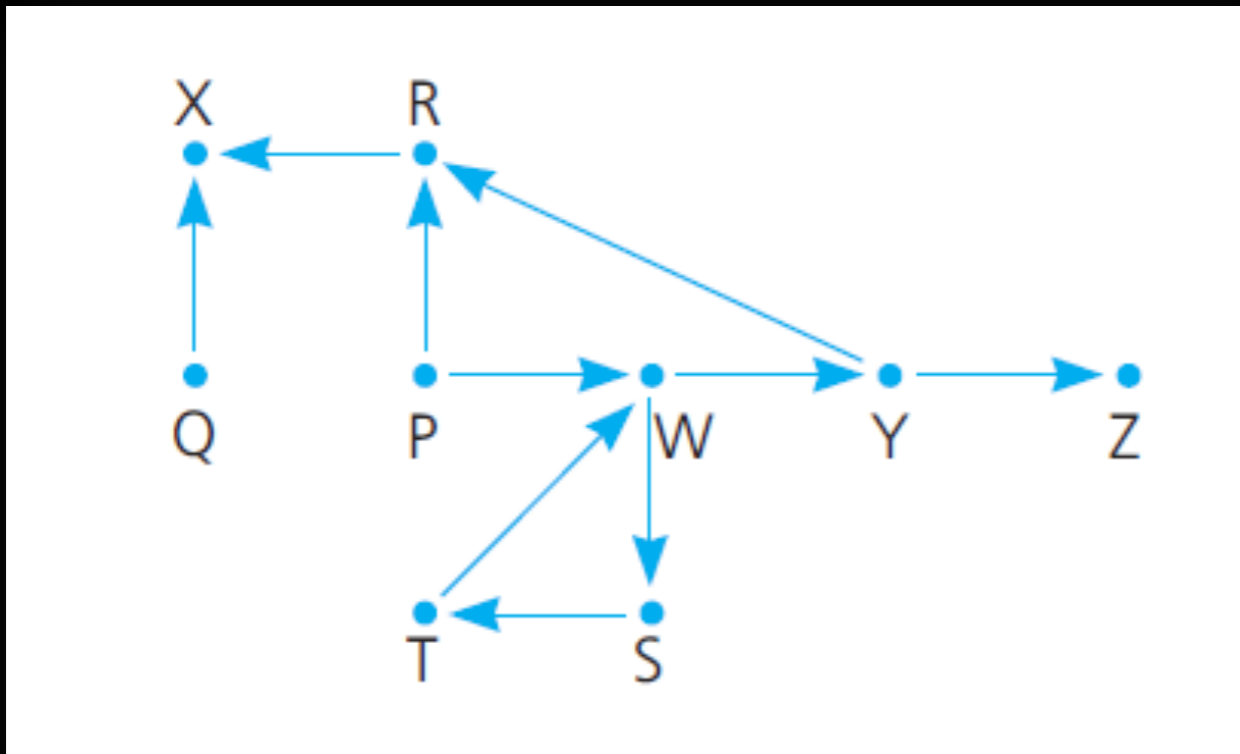
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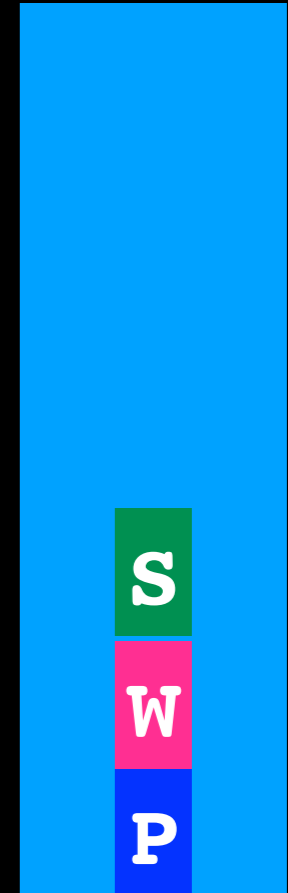
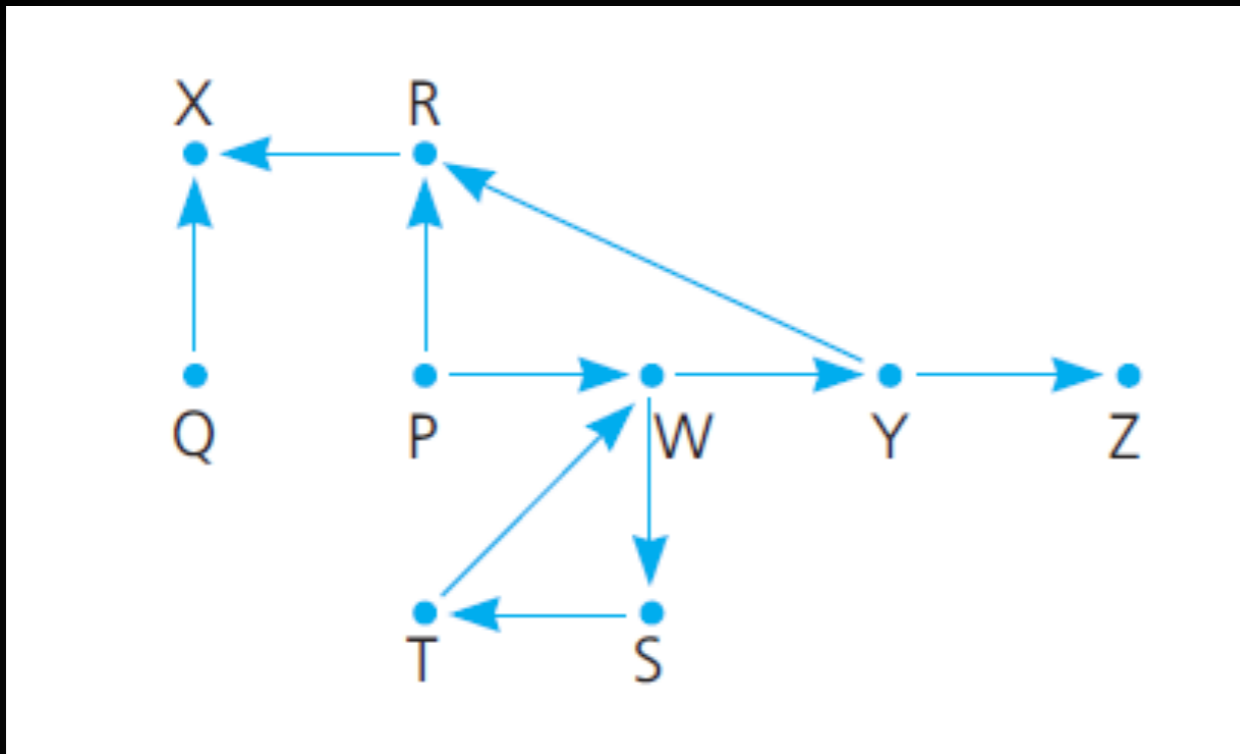
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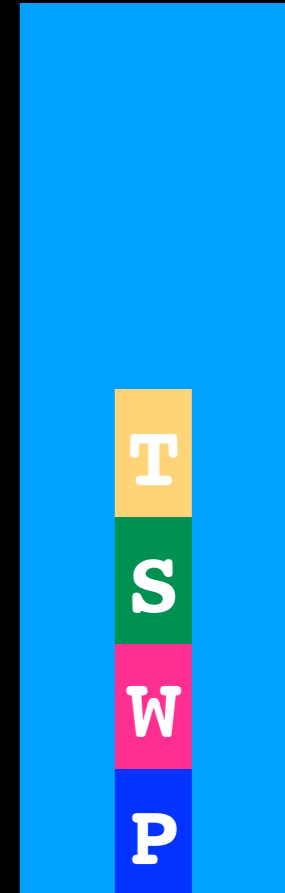
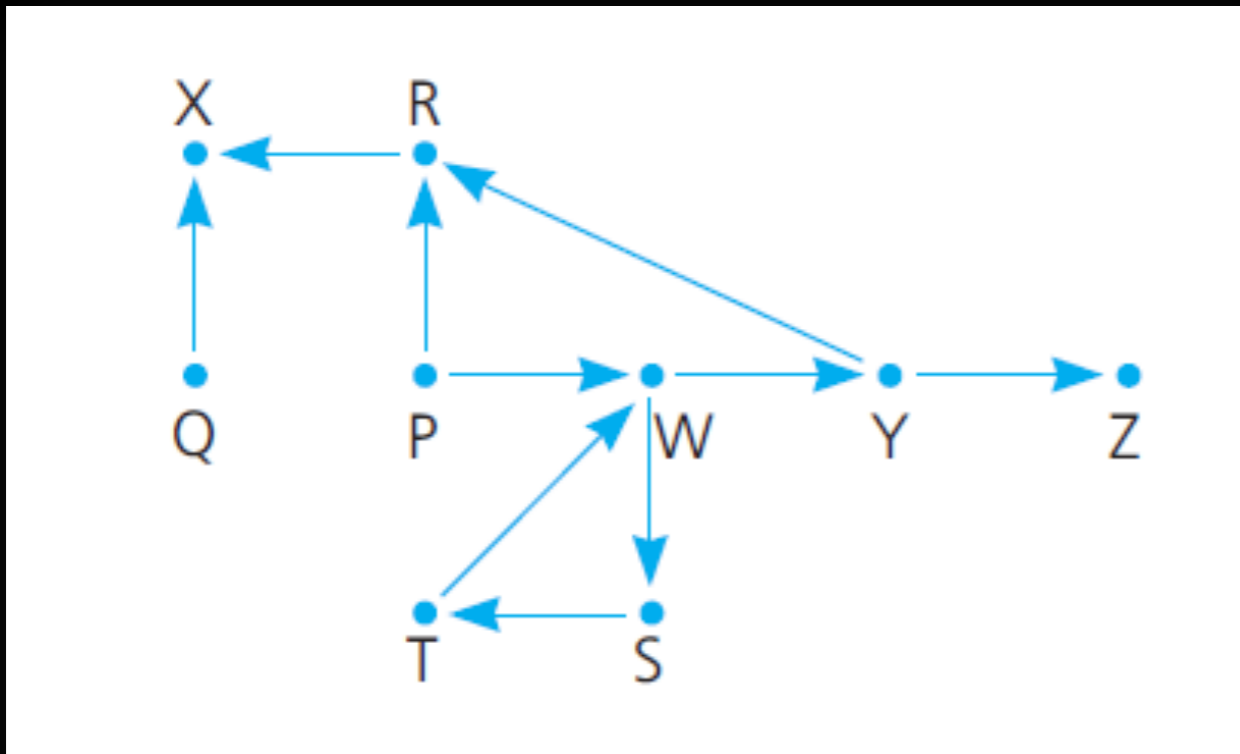
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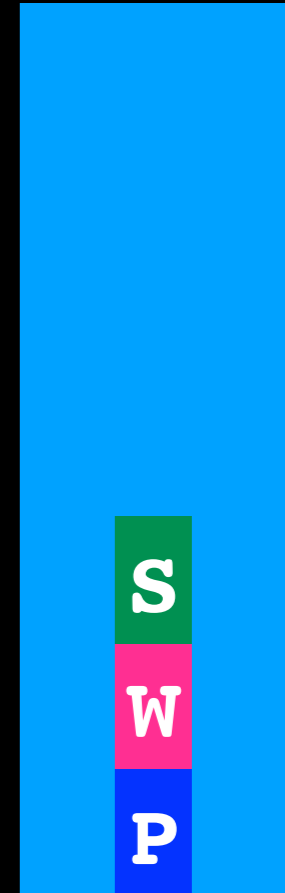
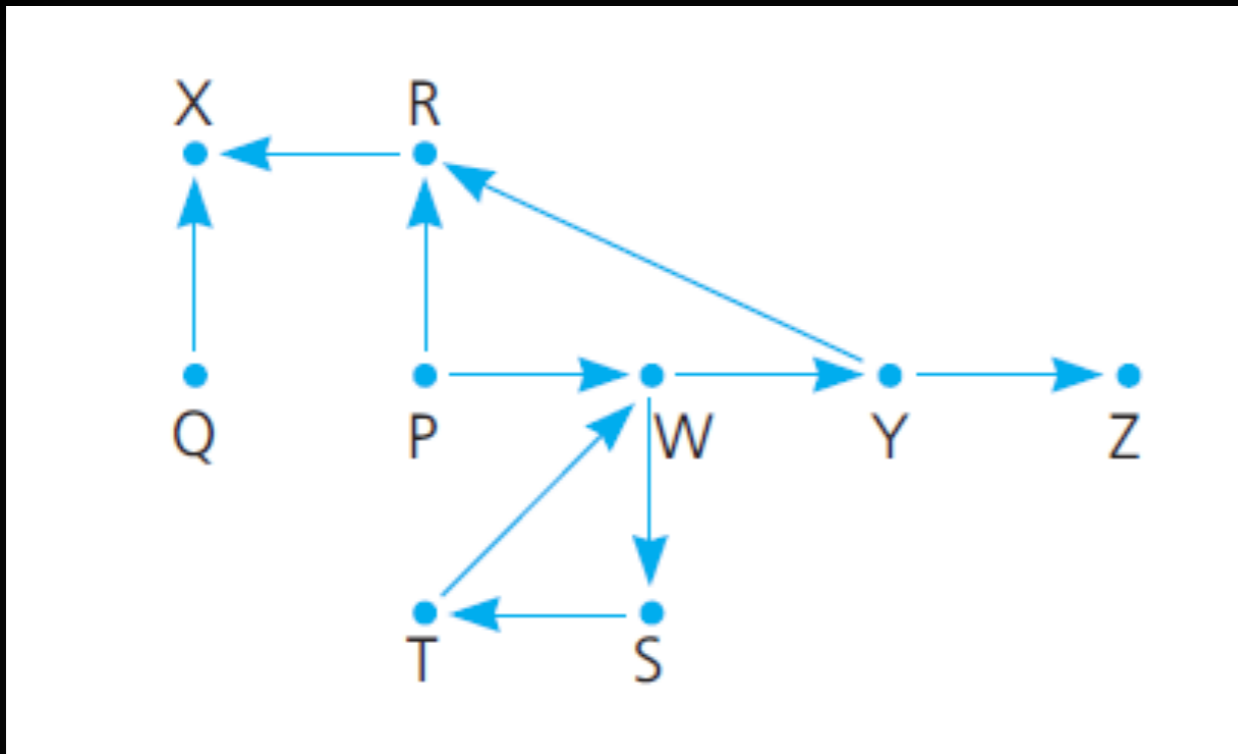
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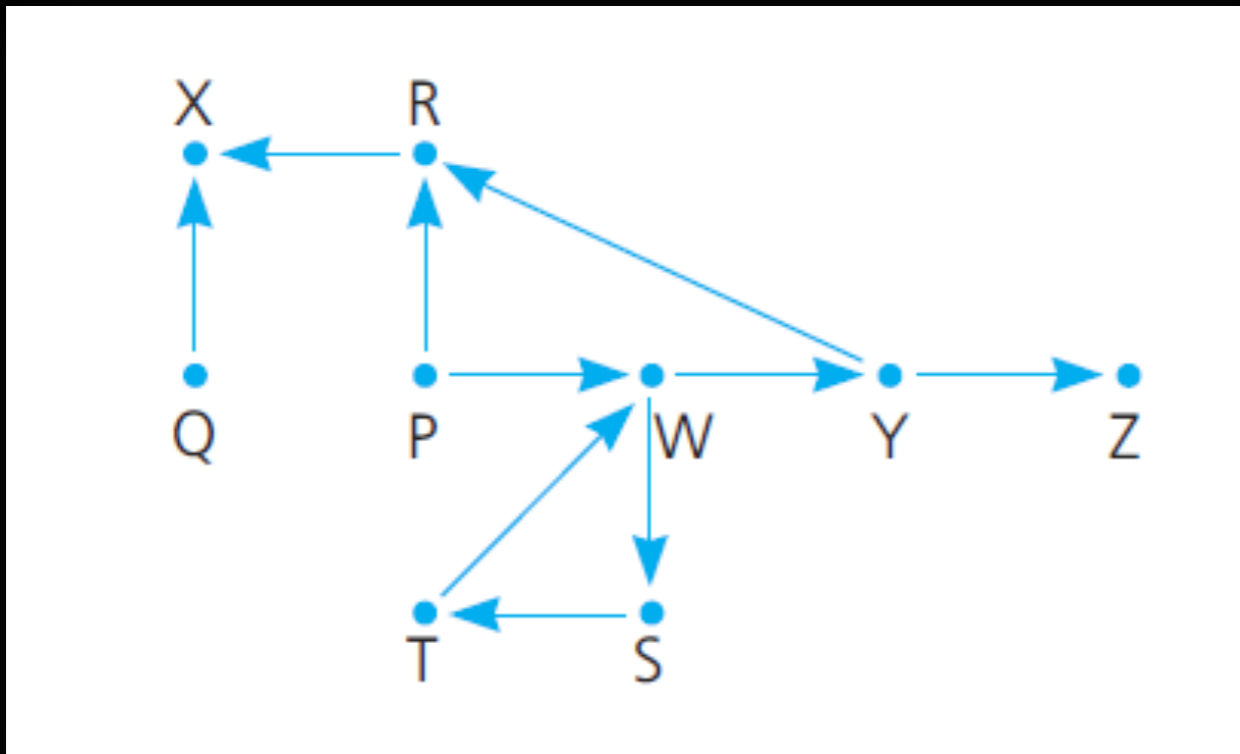
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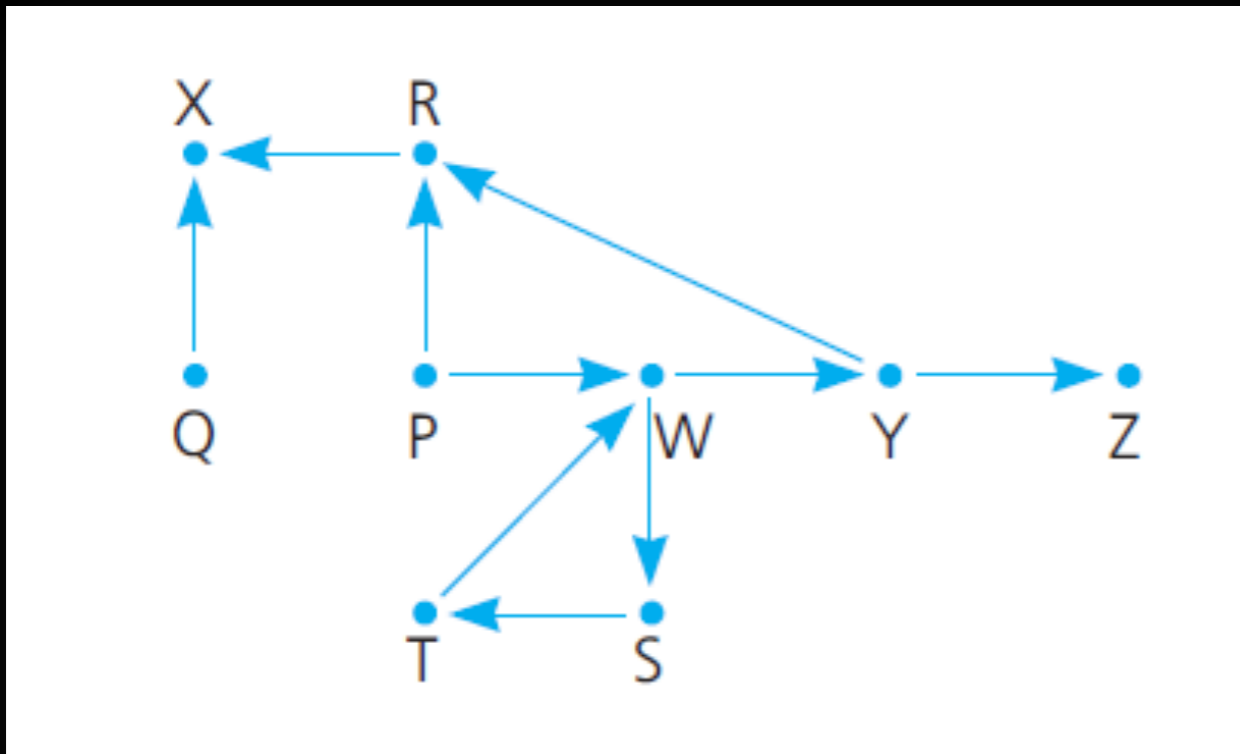
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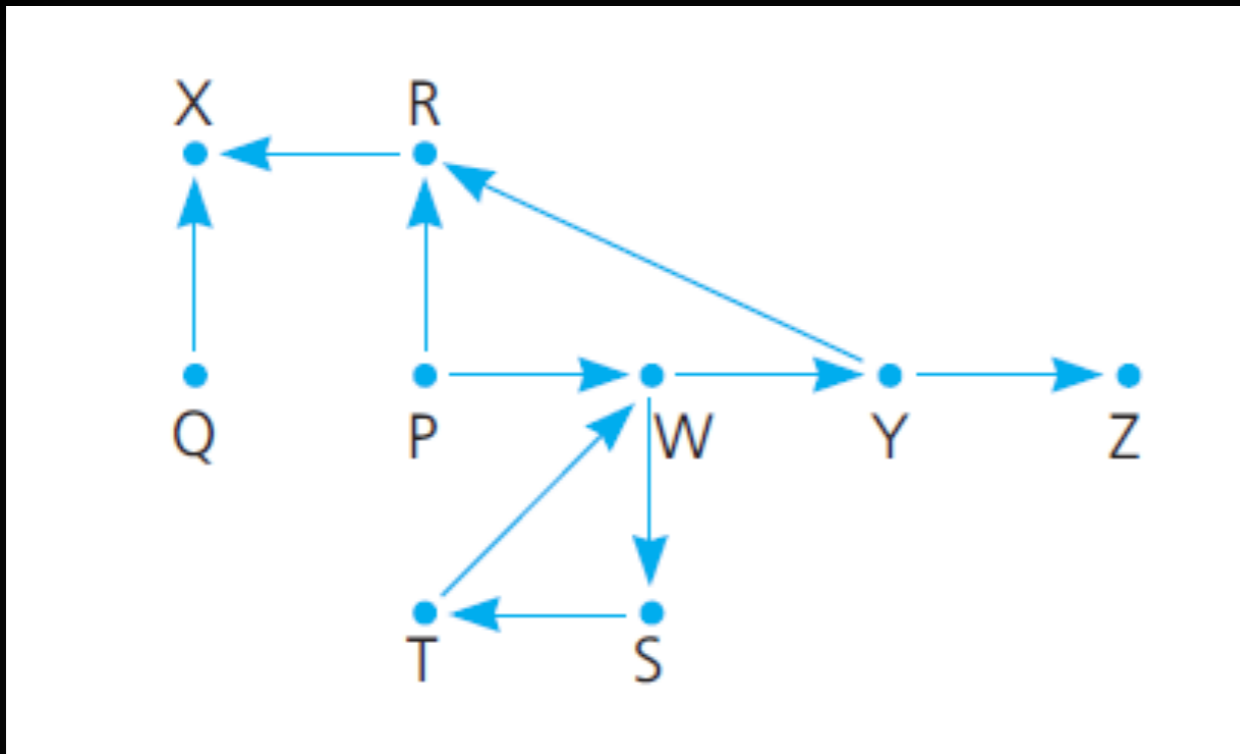
Backtracking

Origin = P , Destination = Z



Backtracking

Origin = P , Destination = Z



Backtracking

```
while(not found flights from origin to destination)
{
  if no flight exists from city on top of stack to
  unvisited destination
    pop the stack //BACKTRACK
  else
  {
    select an unvisited city C accessible from city
    currently at top of stack
    push C on stack
    mark C as visited
  }
}
```

Program Stack and Recursion

Recursion works because function waiting for result/
return from recursive call are on program stack

Order of execution determined by **stack**

More Applications

Balancing anything!

- html tags (e.g `<p>` matches `</p>`)
- parsers in general

Reverse characters in a word or words in a sentence

Undo mechanism for editors or backups

Traversals (graphs / trees)

...

Stack ADT

```
#ifndef STACK_H_
#define STACK_H_

template<class T>
class Stack
{
public:
    Stack();
    void push(const T& new_entry); // adds an element to top of stack
    void pop(); // removes element from top of stack
    T top() const; // returns a copy of element at top of stack
    int size() const; // returns the number of elements in the stack
    bool isEmpty() const; // returns true if no elements on stack false otherwise

private:
    //implementation details here

}; //end Stack

#include "Stack.cpp"
#endif // STACK_H_`
```


Abstract Data Types

Bag

List

Stack

Queue

Queue

An ADT representing a waiting line

Objects can be **enqueued** to the back of the line
or **dequeued** from the front of the line



34

Queue

An ADT representing a waiting line

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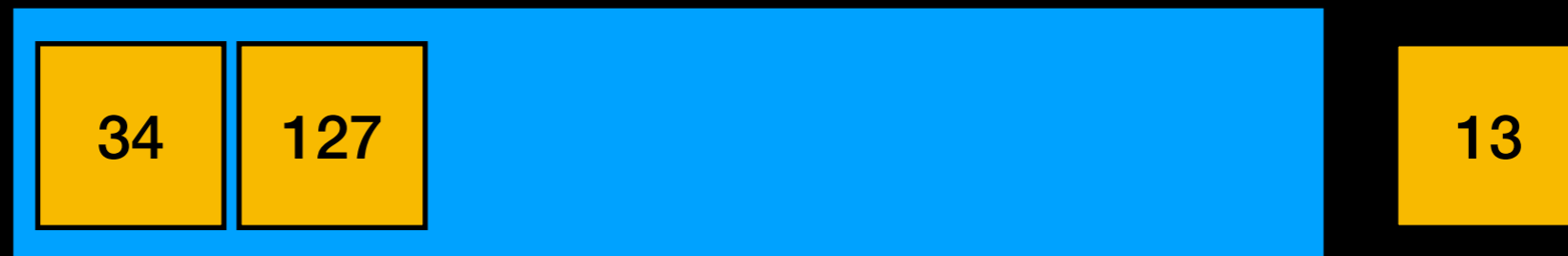


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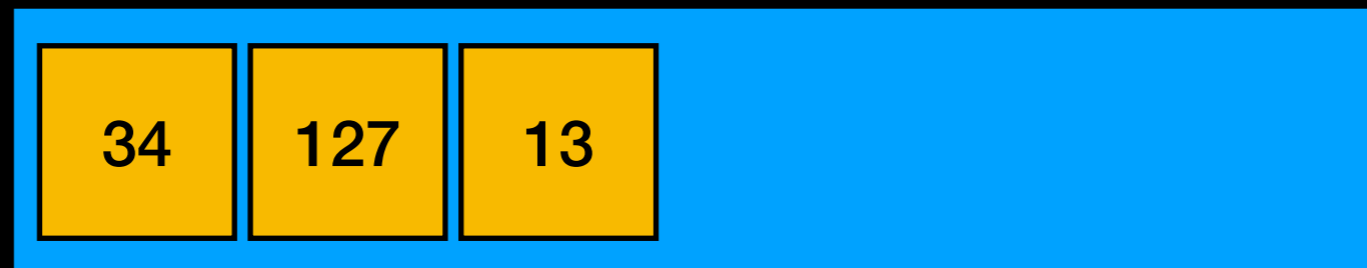


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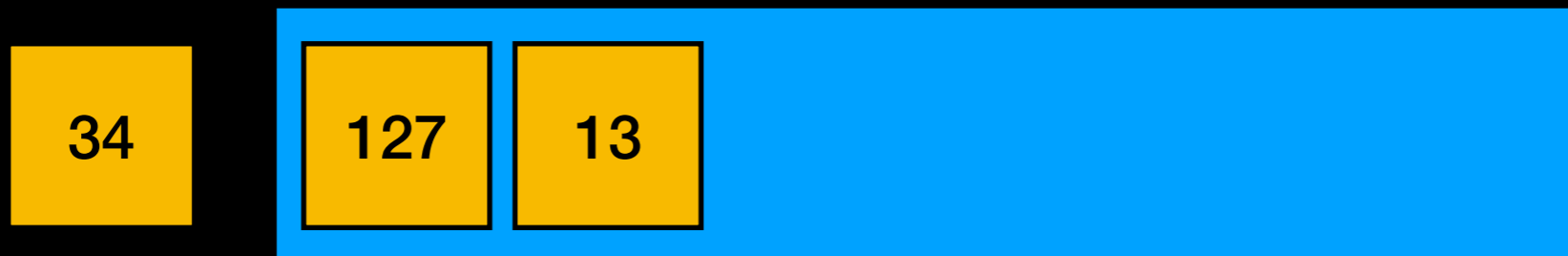


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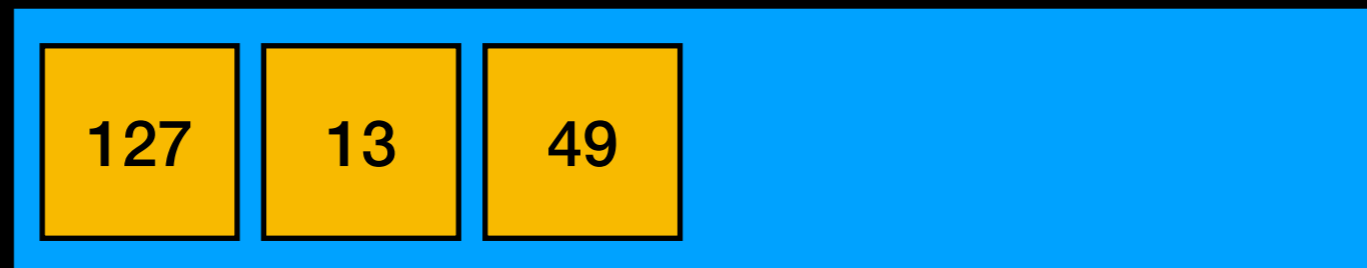


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FIFO: First In First Out

Only front of queue is accessible (**front**), no other objects in the queue are visible

Queue Applications

Generating all substrings

Any waiting queue

- Print jobs
- OS scheduling processes with equal priority
- Messages between asynchronous processes

...

Queue Applications

Generating all substrings

Any waiting queue

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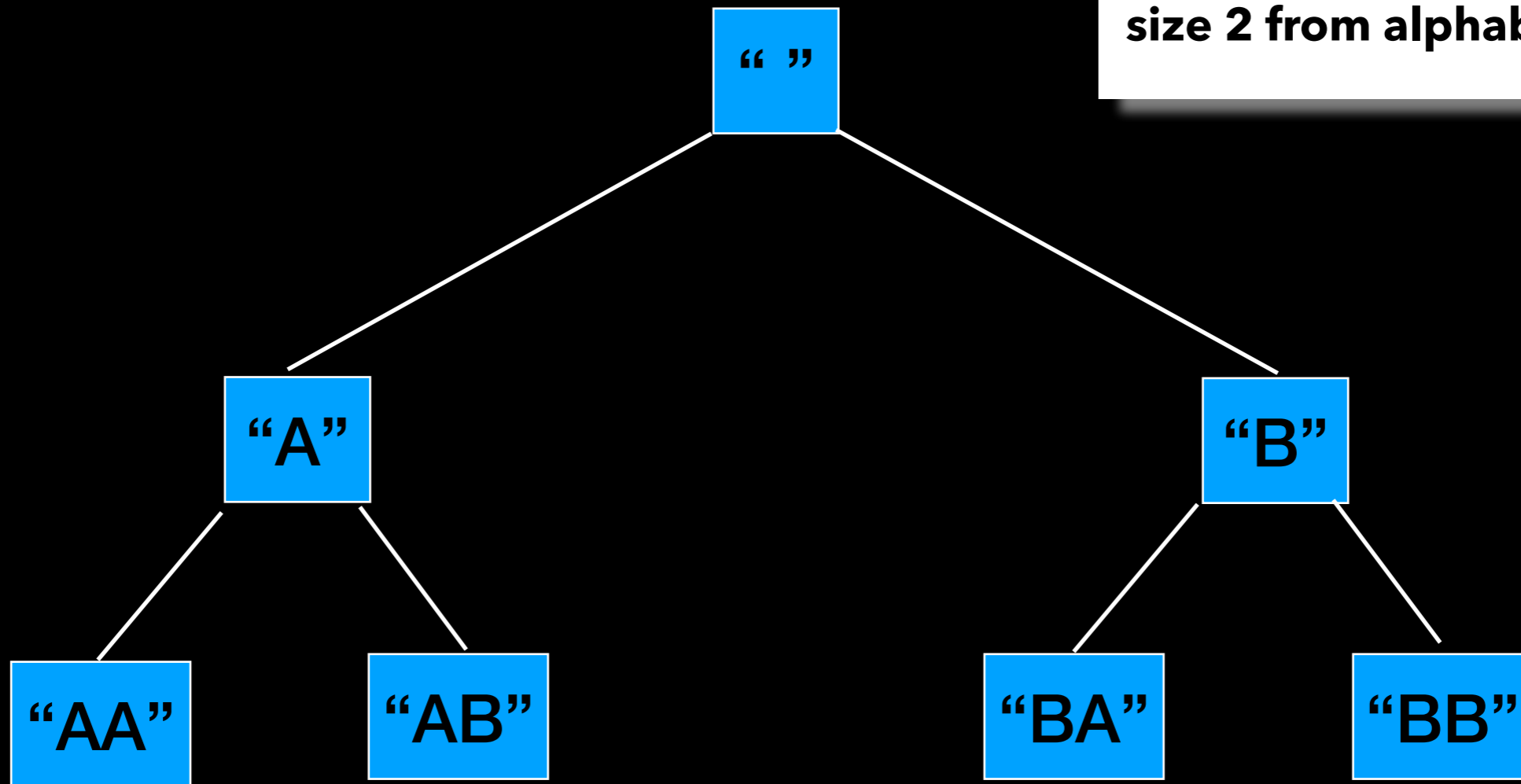
Generating all substrings

Generate all possible strings **up to** some fixed length **n** **with repetition (same character included multiple times)**

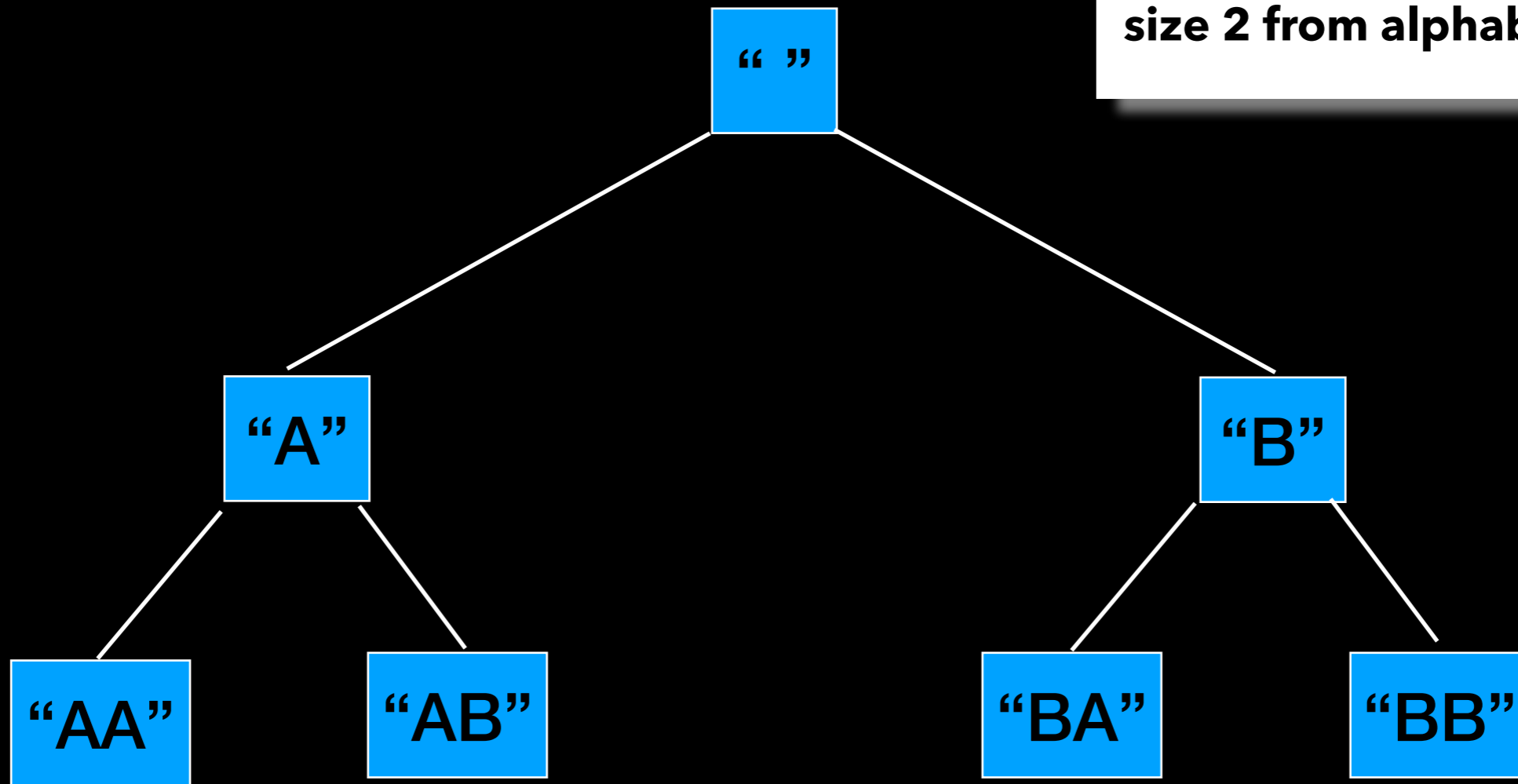
How might we do it with a queue?

Example simplified to $n = 2$ and only letters A and B

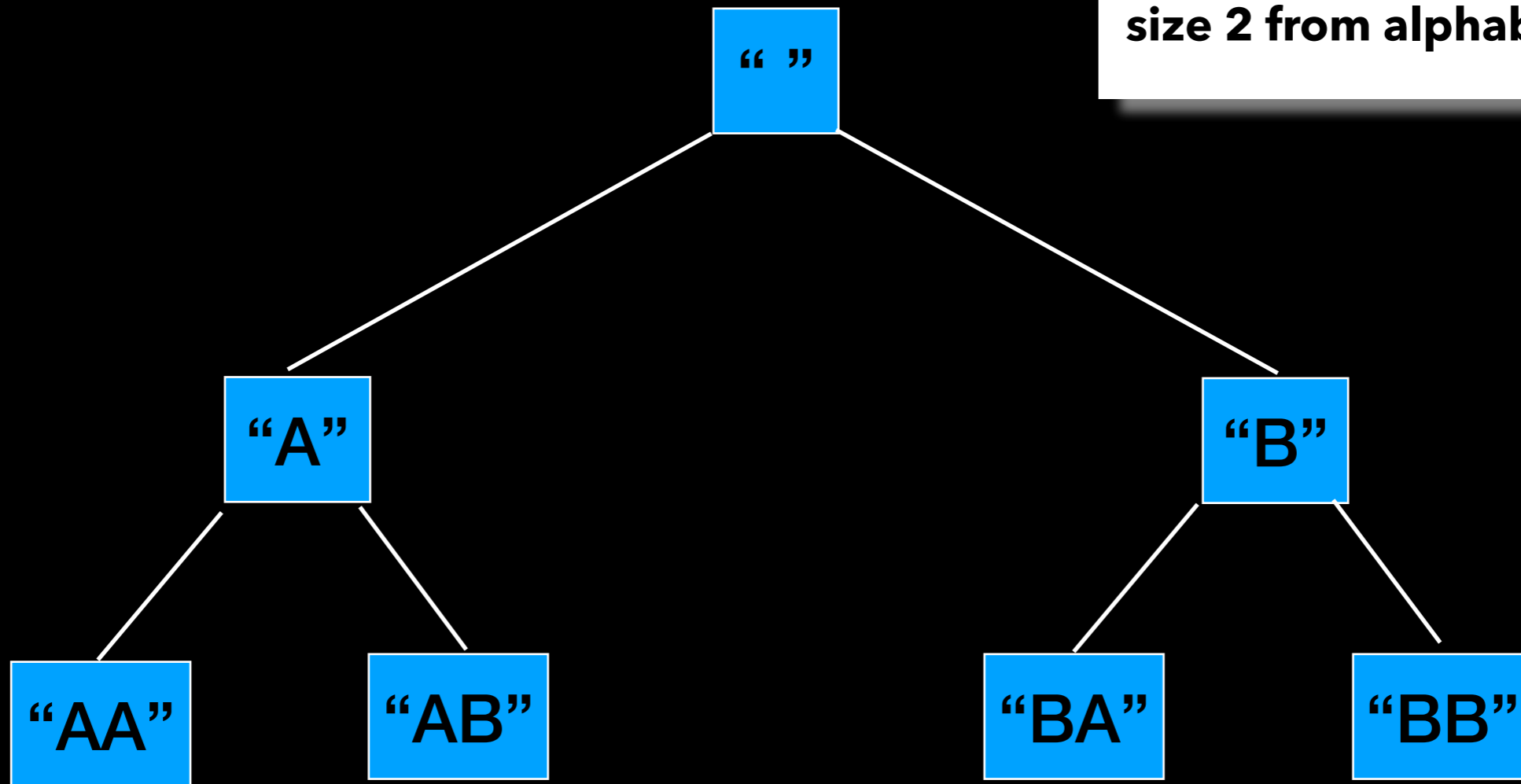
Generate all substrings of size 2 from alphabet {'A', 'B'}



Generate all substrings of size 2 from alphabet {'A', 'B'}



Generate all substrings of size 2 from alphabet {'A', 'B'}

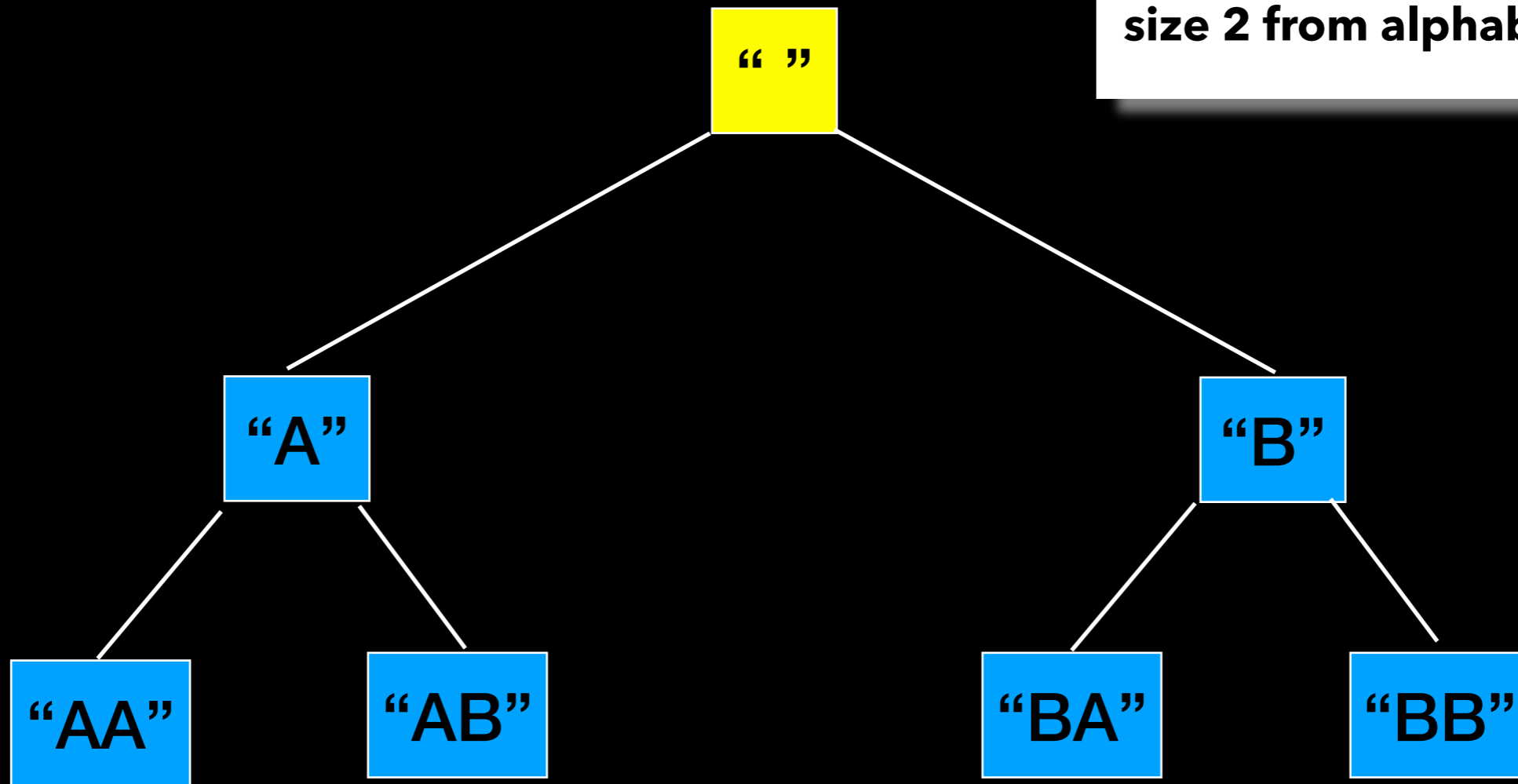


“ ”



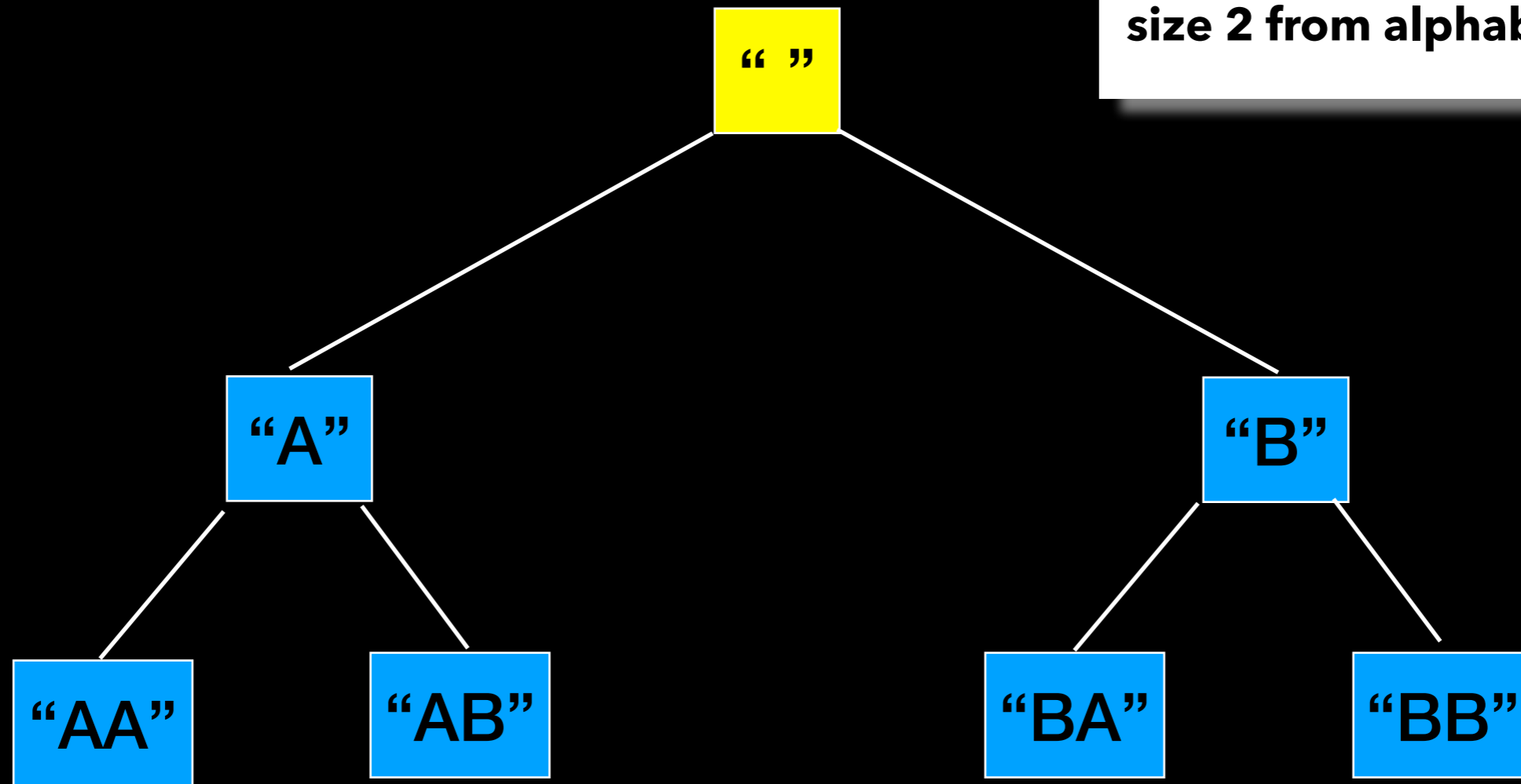
{ "" }

Generate all substrings of size 2 from alphabet {'A', 'B'}



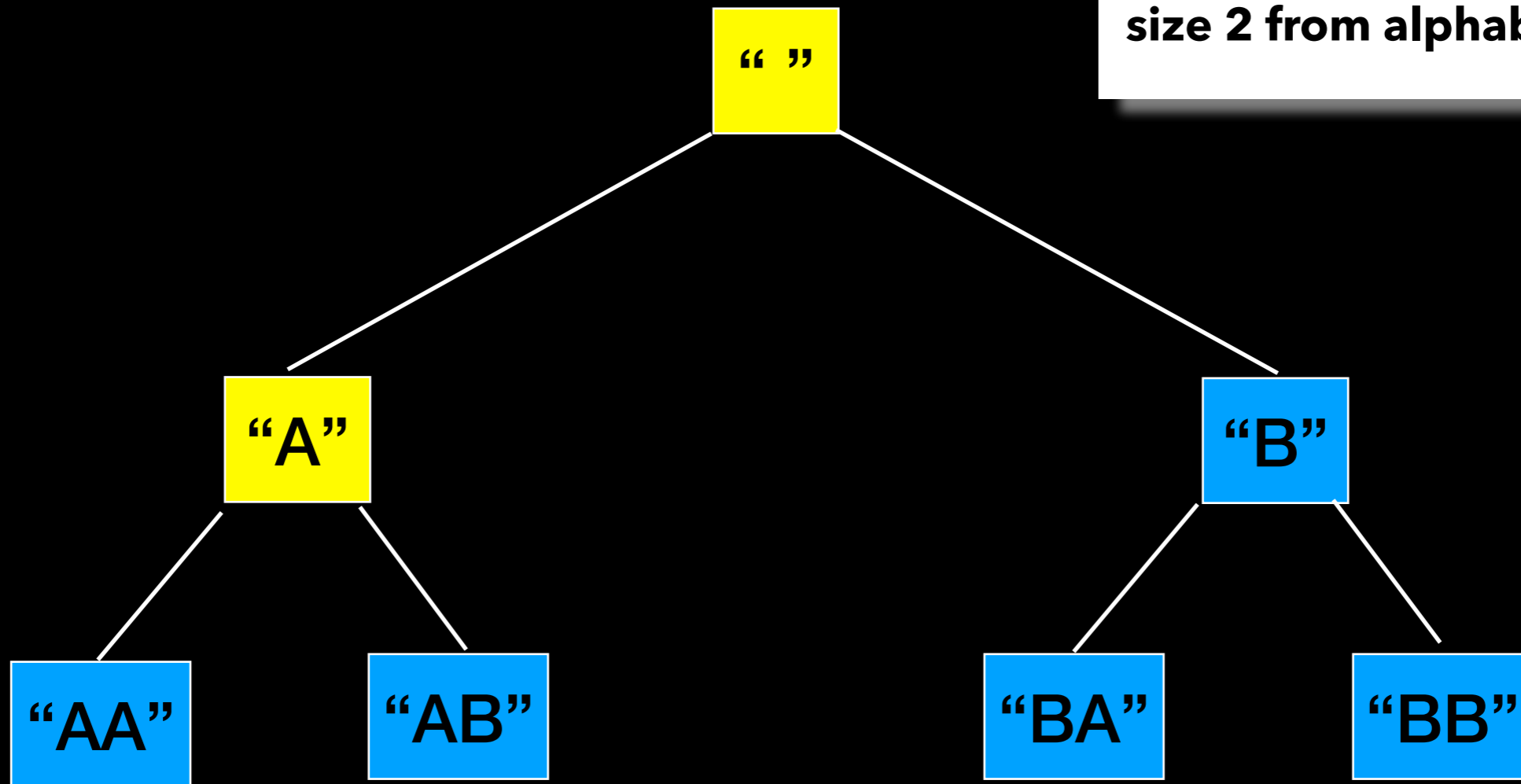
{ "" }

Generate all substrings of size 2 from alphabet {'A', 'B'}



{ "", "A" }

Generate all substrings of size 2 from alphabet {'A', 'B'}

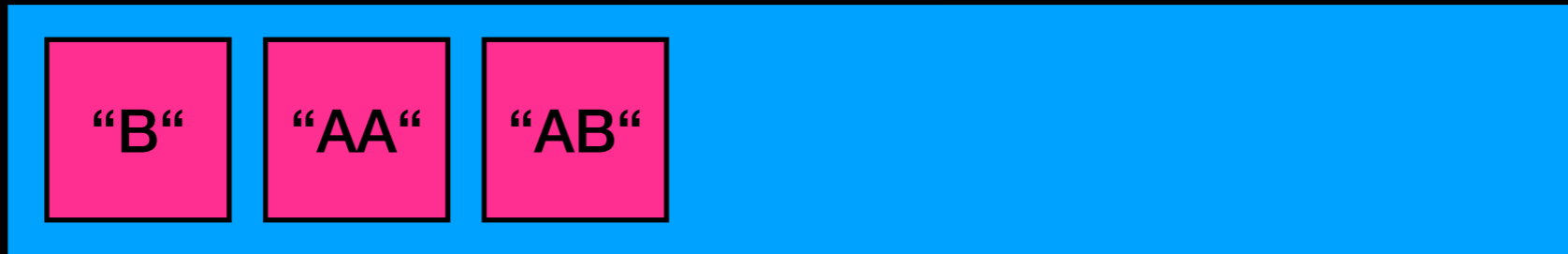
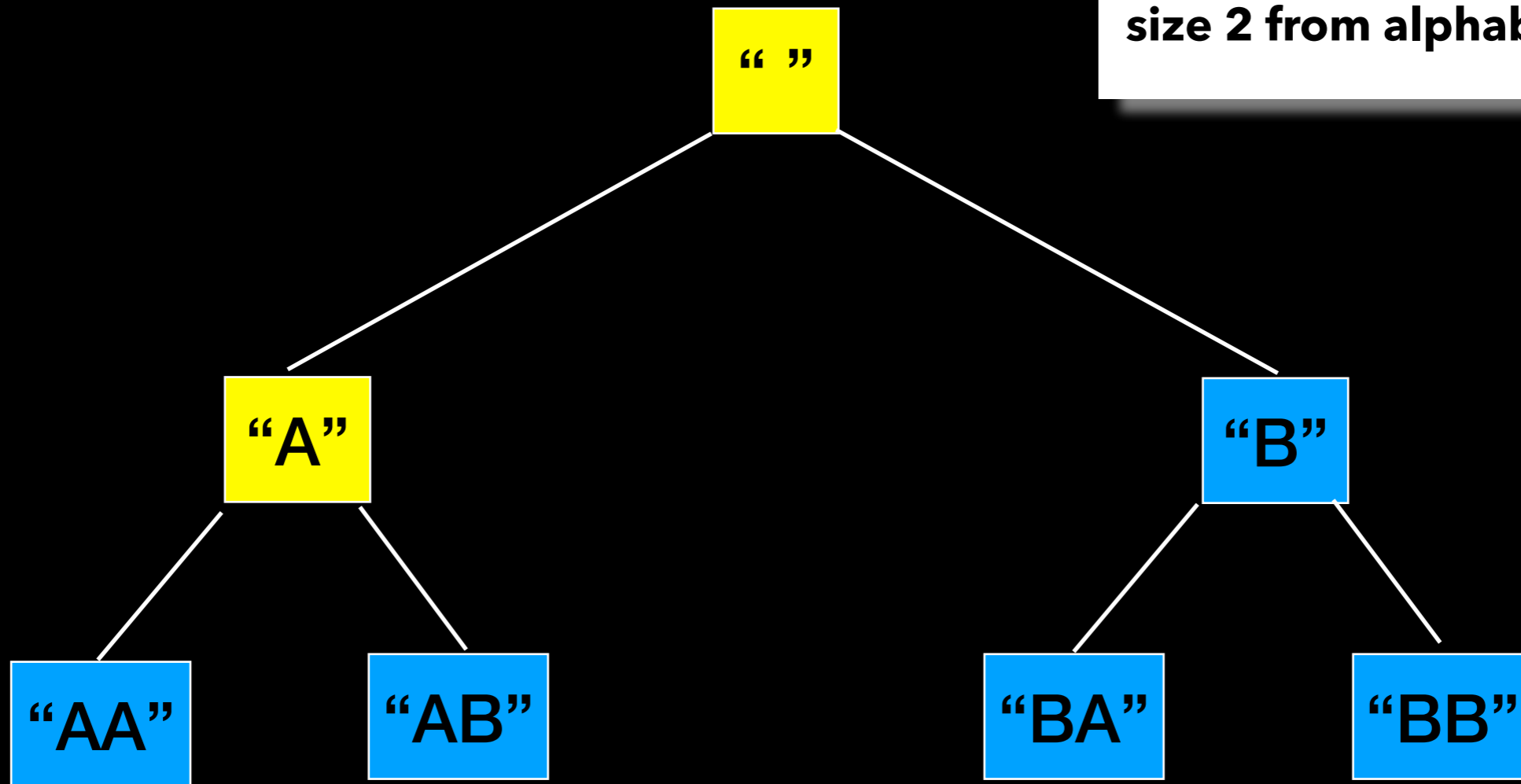


“A” “AA” “AB”

“B”

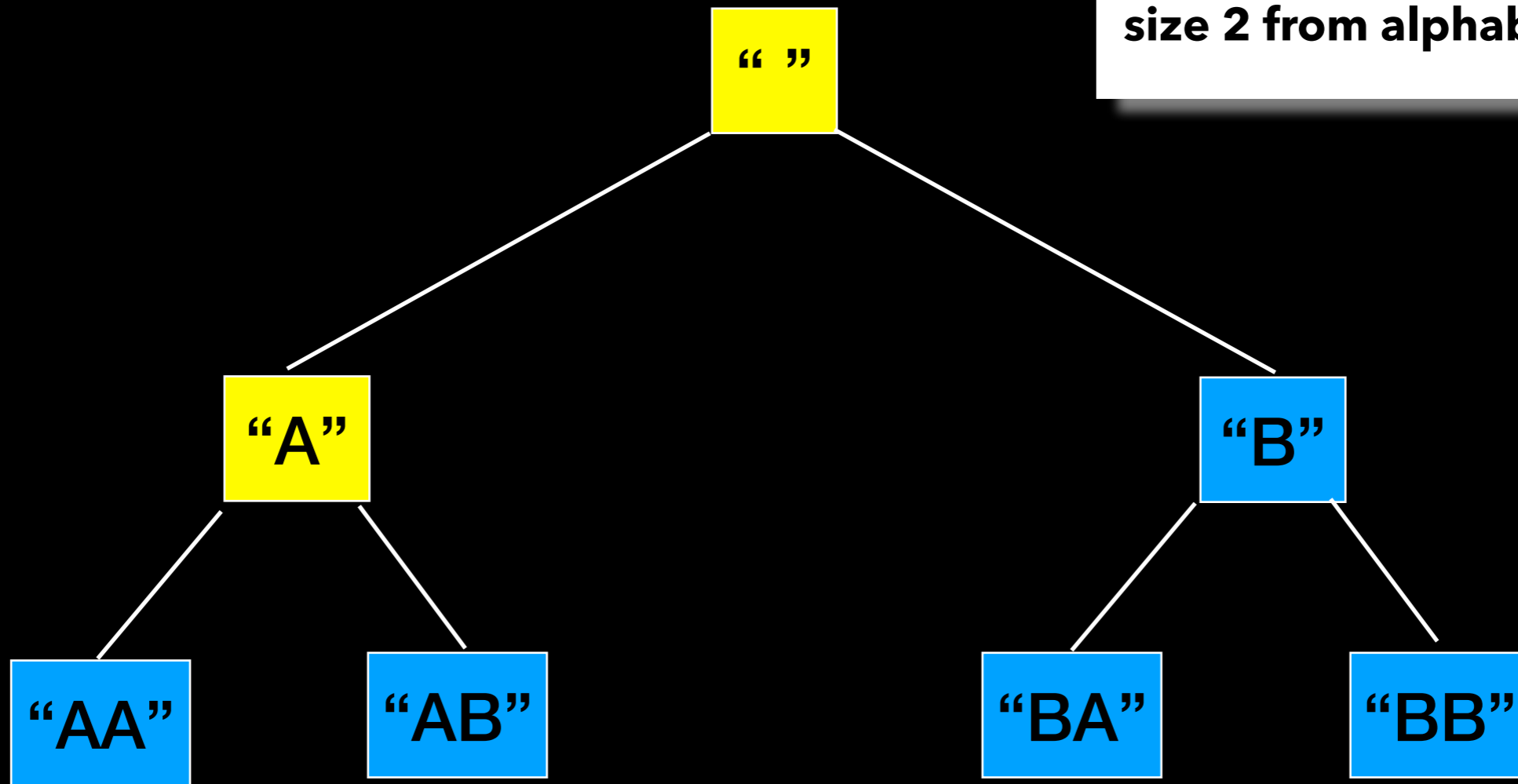
{ "", "A" }

Generate all substrings of size 2 from alphabet {'A', 'B'}



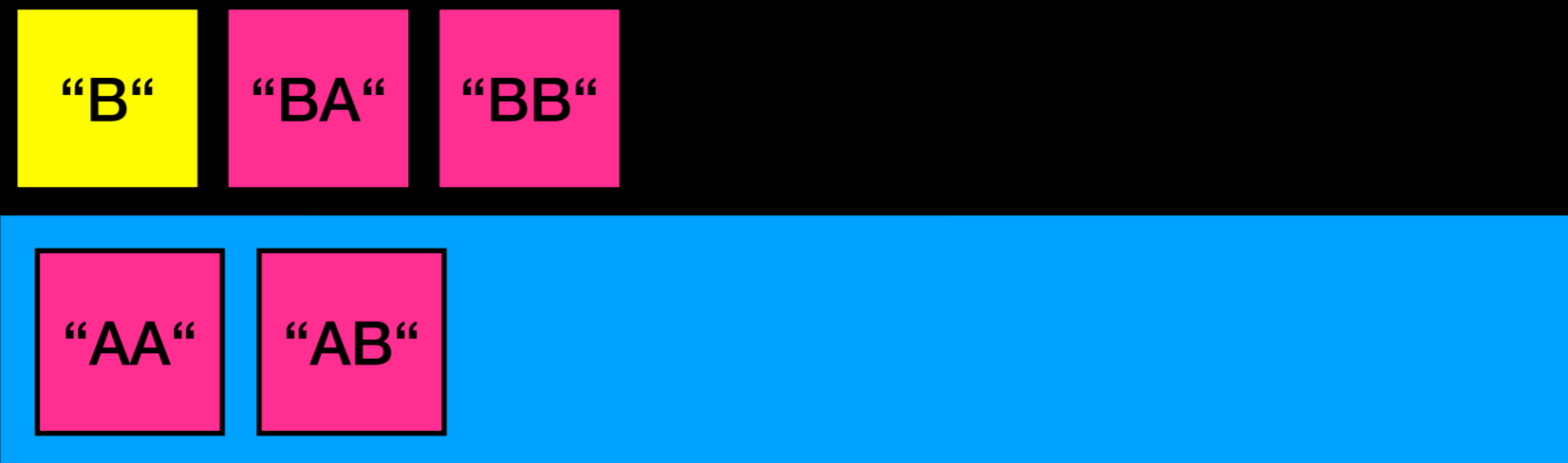
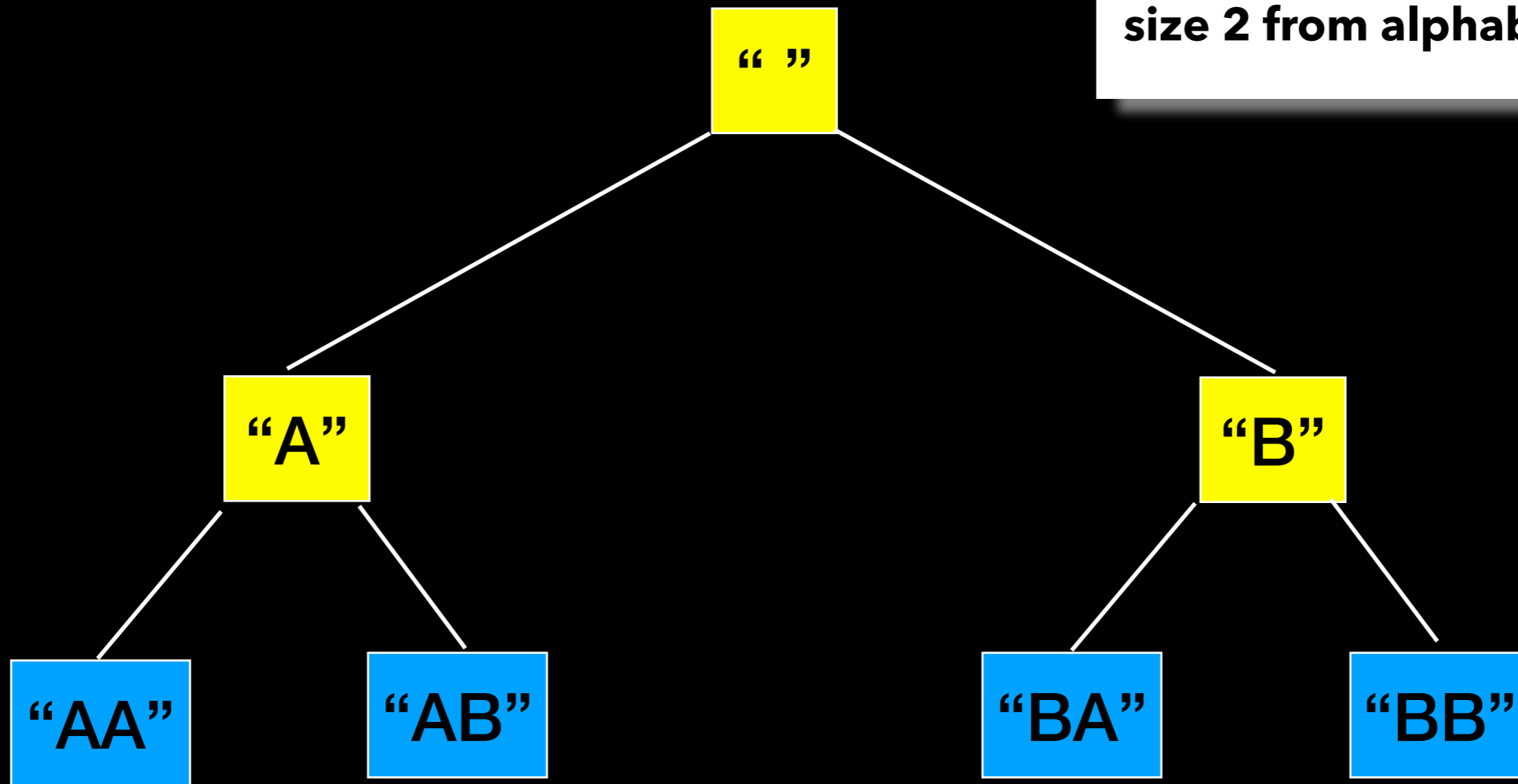
{ "", "A" }

Generate all substrings of size 2 from alphabet {'A', 'B'}



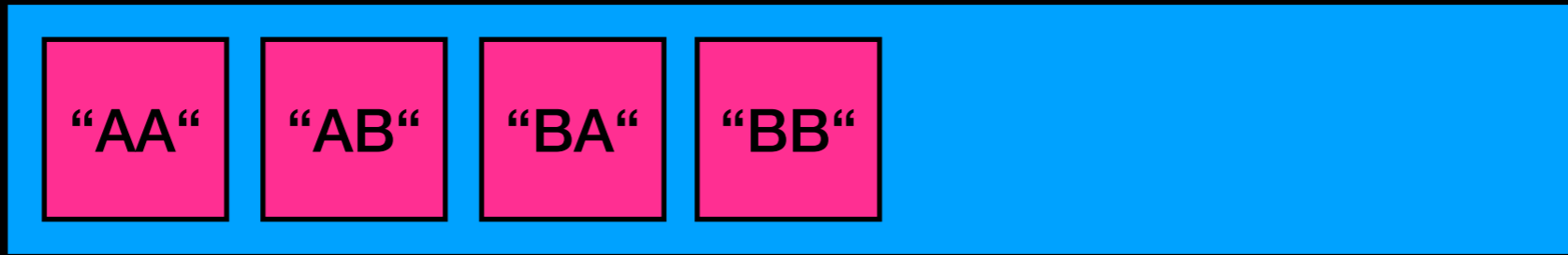
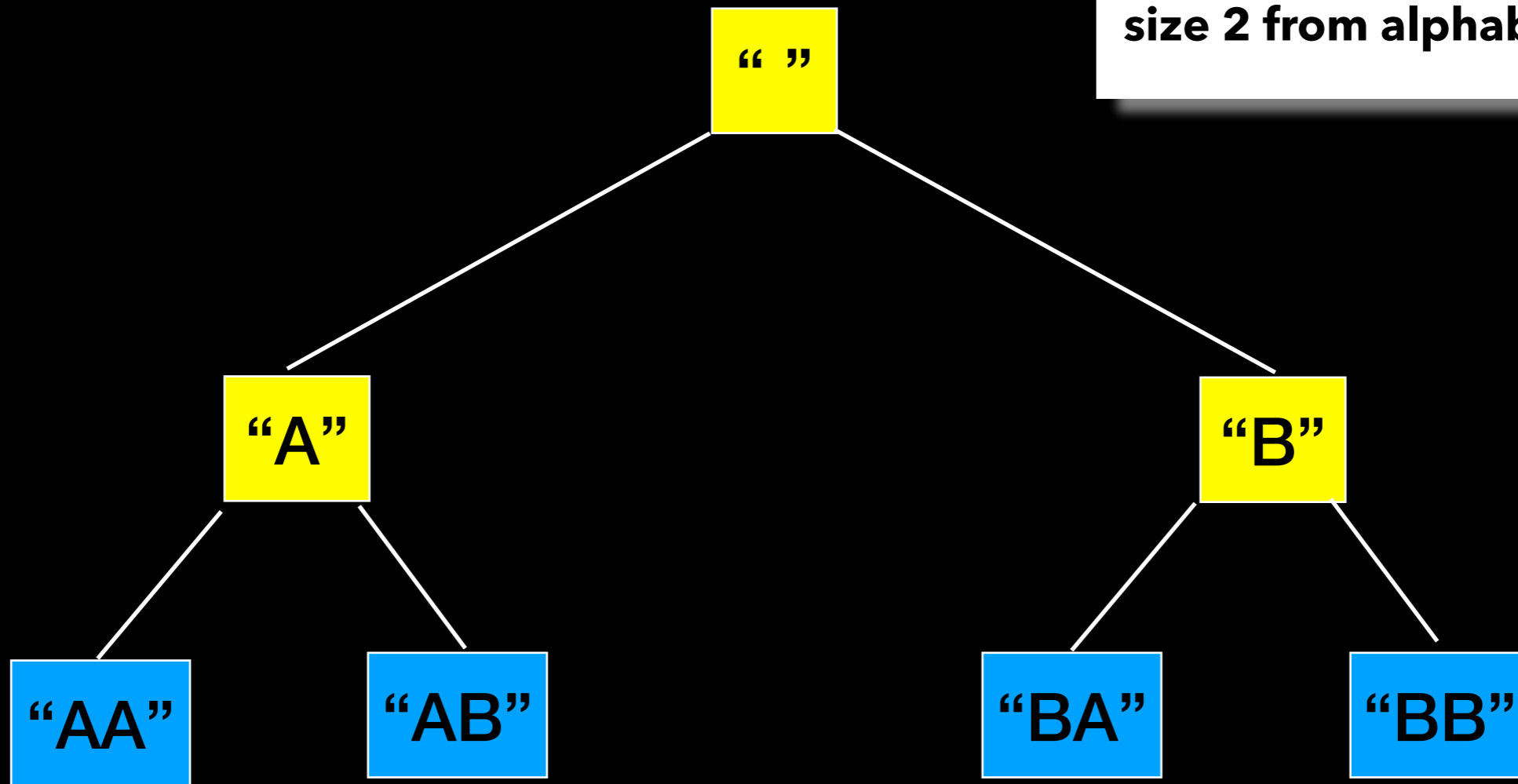
{ "", "A", "B" }

Generate all substrings of size 2 from alphabet {'A', 'B'}



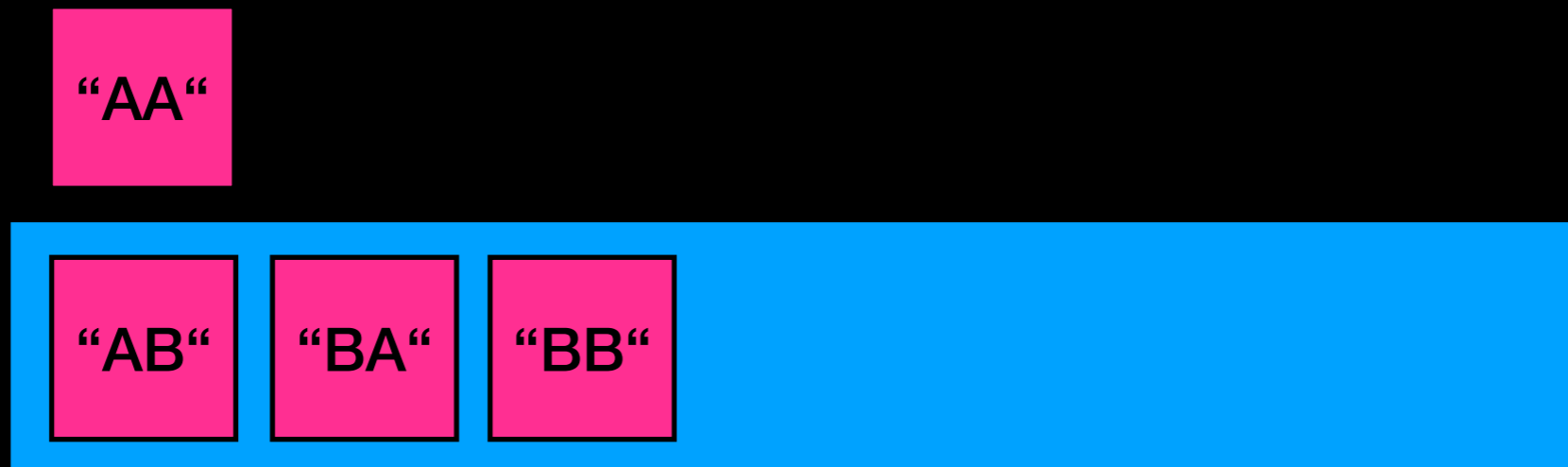
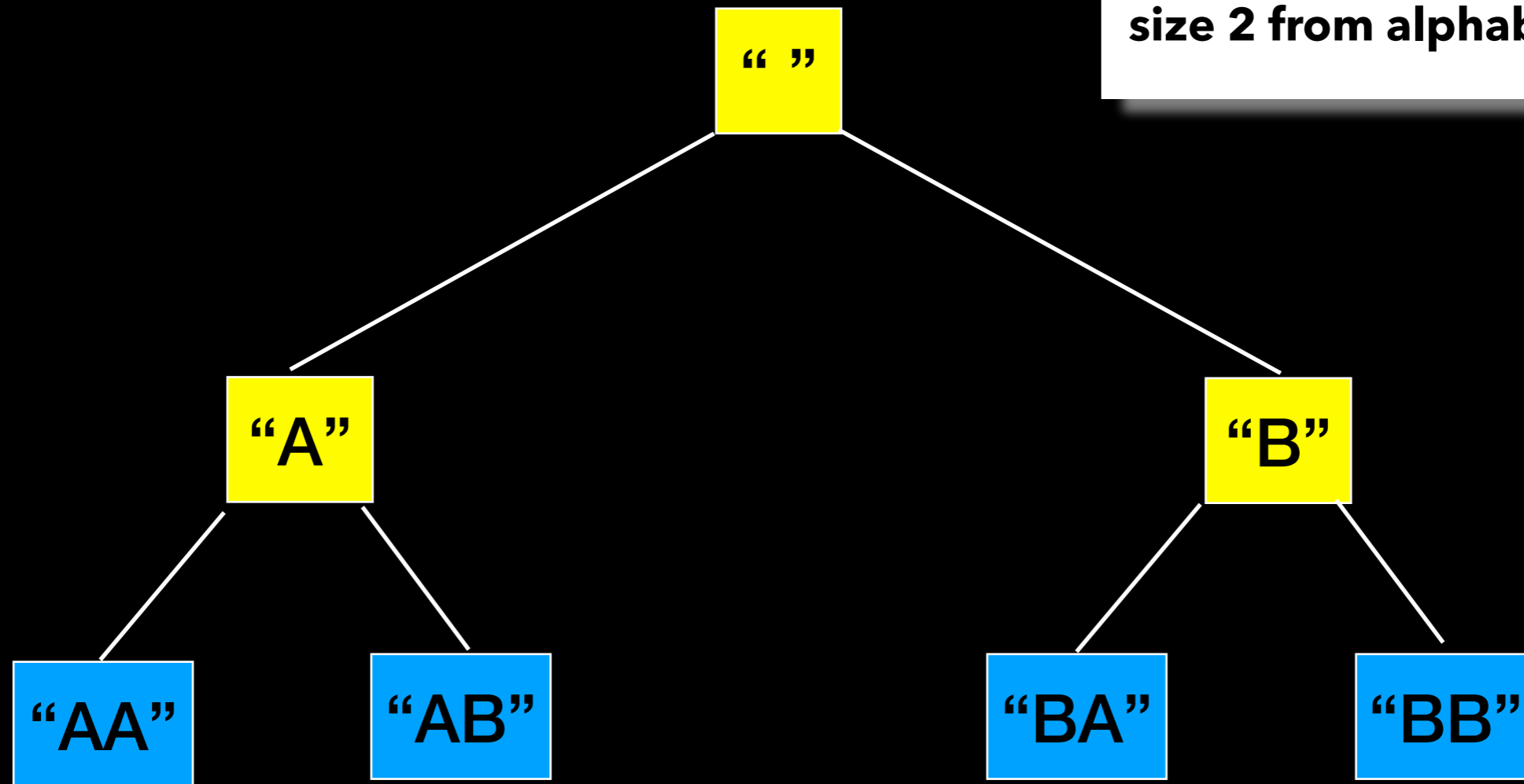
{ "", "A", "B" }

Generate all substrings of size 2 from alphabet {'A', 'B'}



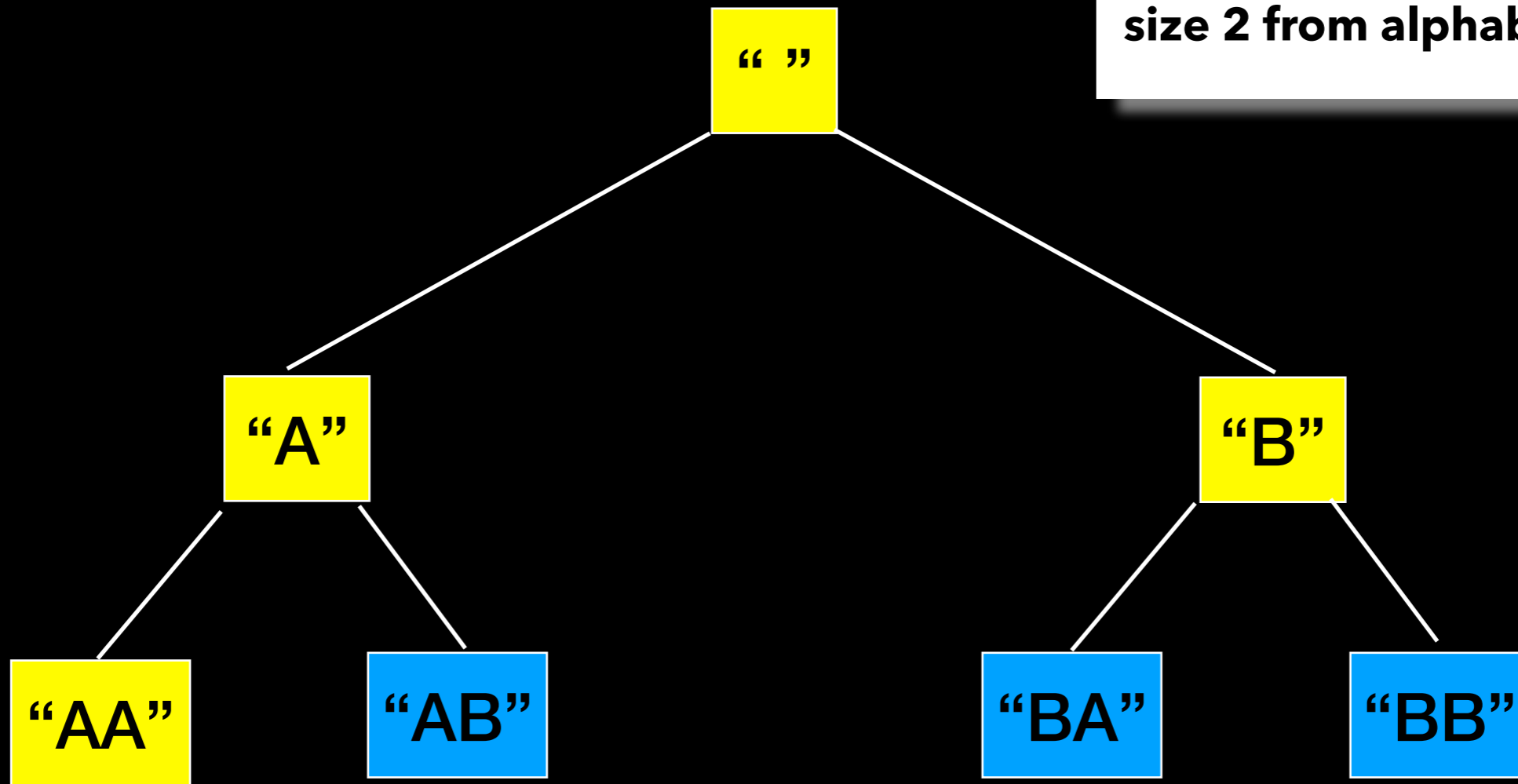
{ "", "A", "B" }

Generate all substrings of size 2 from alphabet {'A', 'B'}

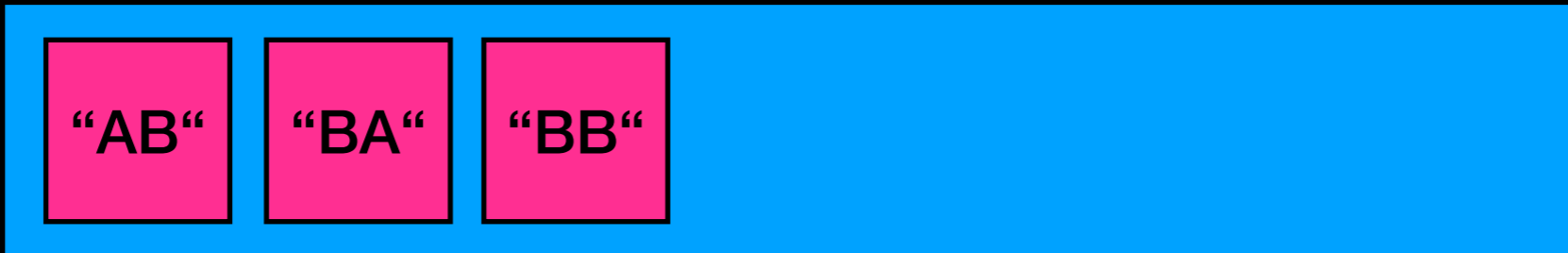


{ "", "A", "B", "AA" }

Generate all substrings of size 2 from alphabet {'A', 'B'}

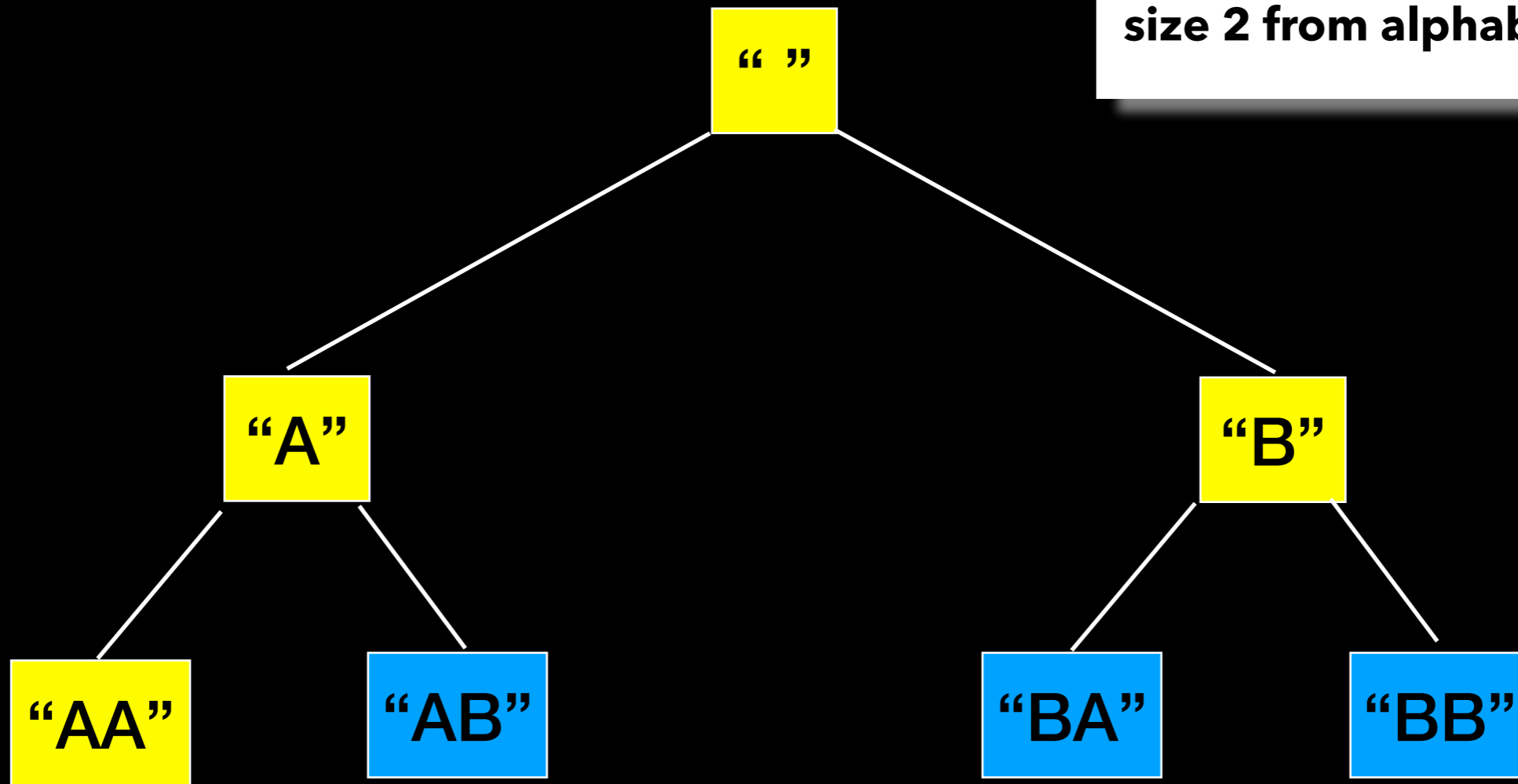


“AA“



{ "", "A", "B", "AA" }

Generate all substrings of size 2 from alphabet {'A', 'B'}

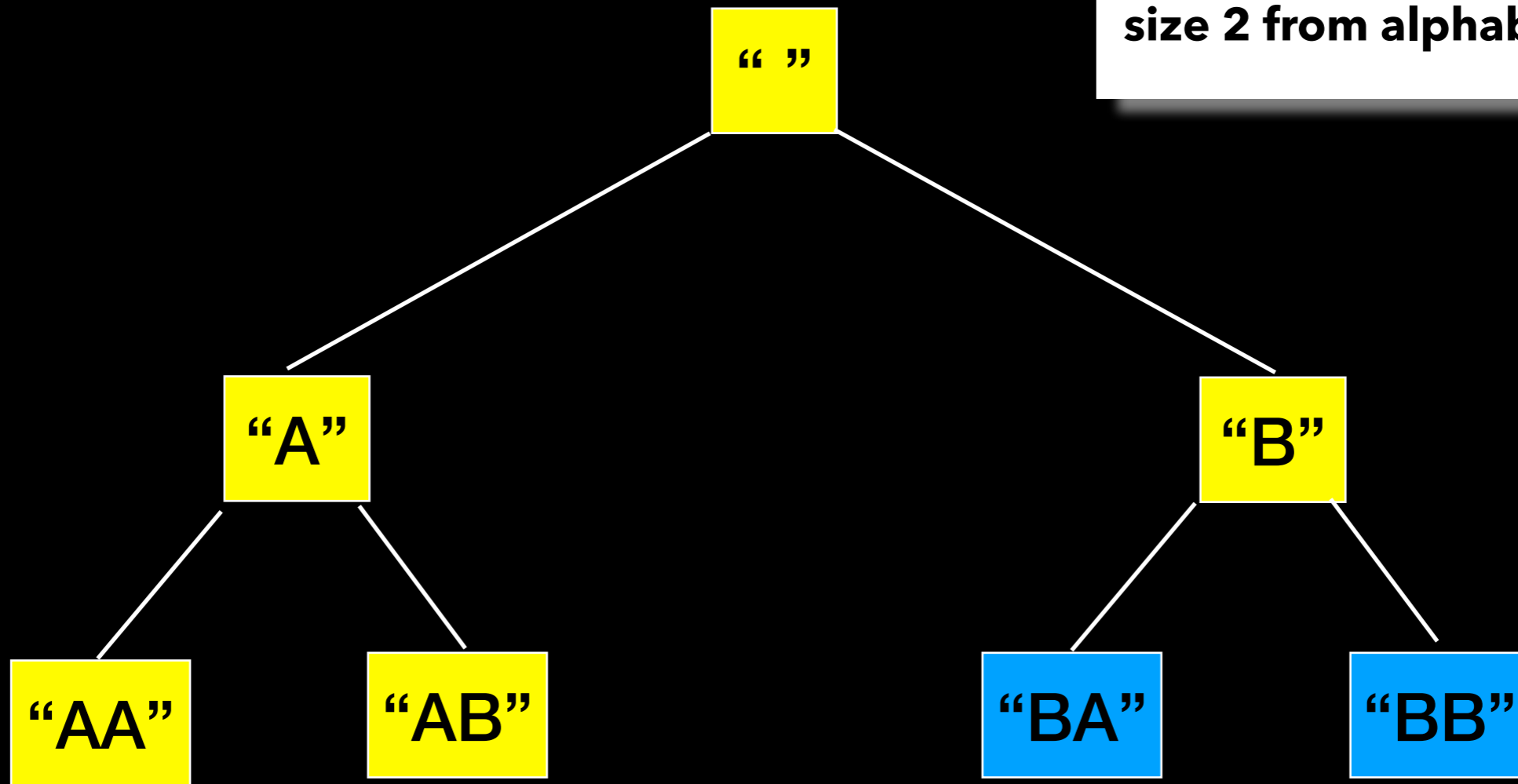


“AB“



{ "", "A", "B", "AA", "AB" }

Generate all substrings of size 2 from alphabet {'A', 'B'}

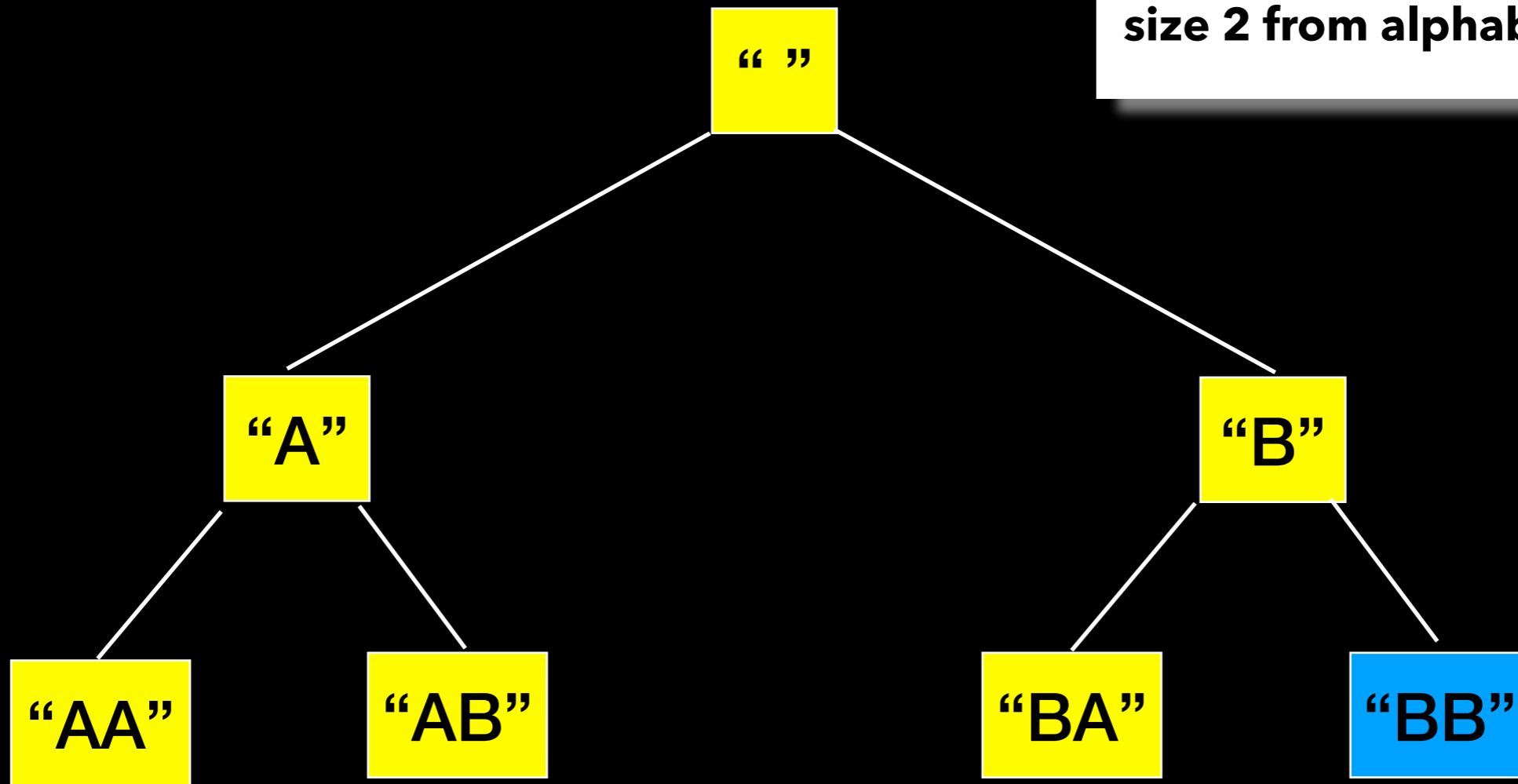


"AB"



{ "", "A", "B", "AA", "AB", "BA" }

Generate all substrings of size 2 from alphabet {'A', 'B'}

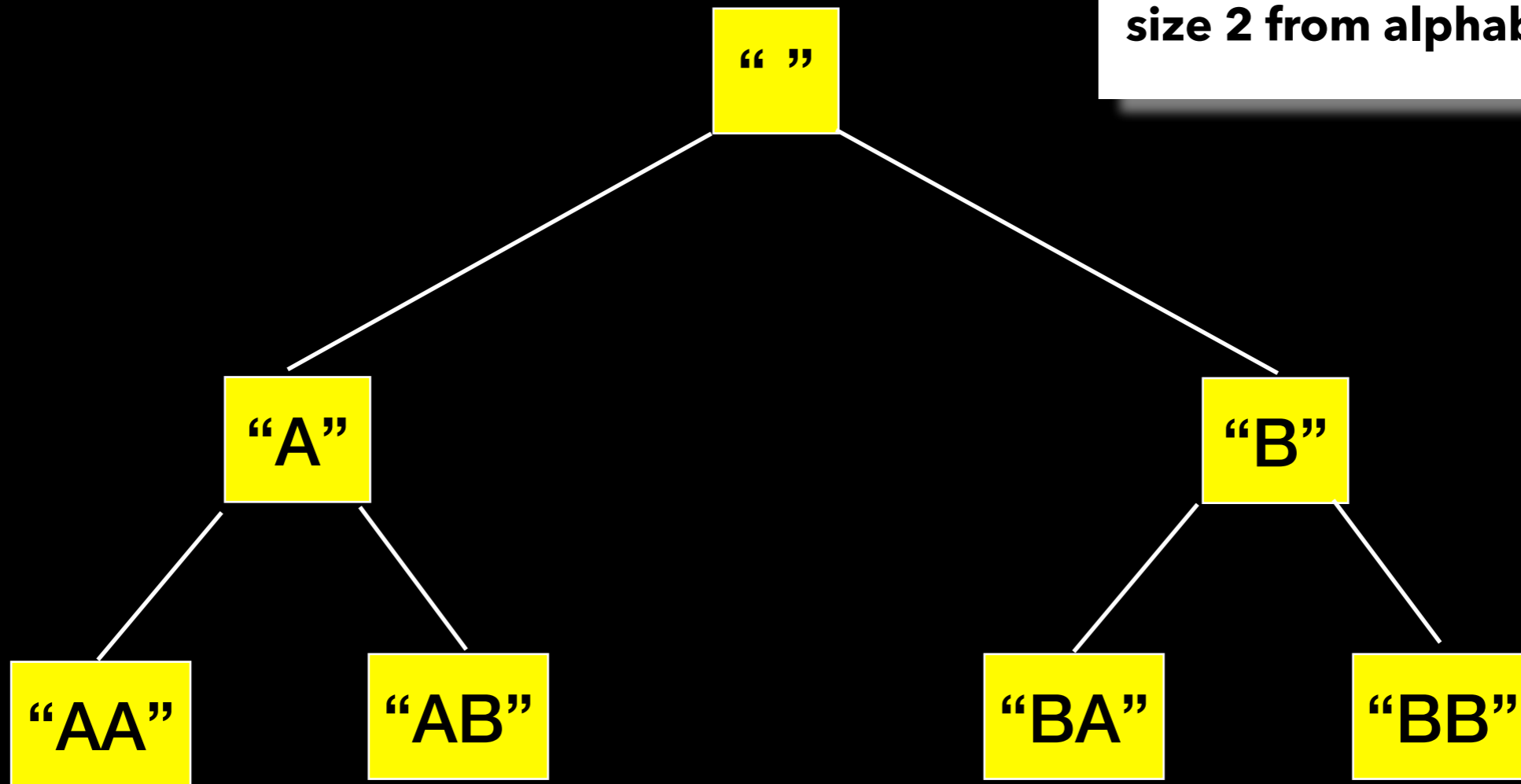


“BA“

“BB“

{ "", "A", "B", "AA", "AB", "BA", "BB" }

Generate all substrings of size 2 from alphabet {'A', 'B'}

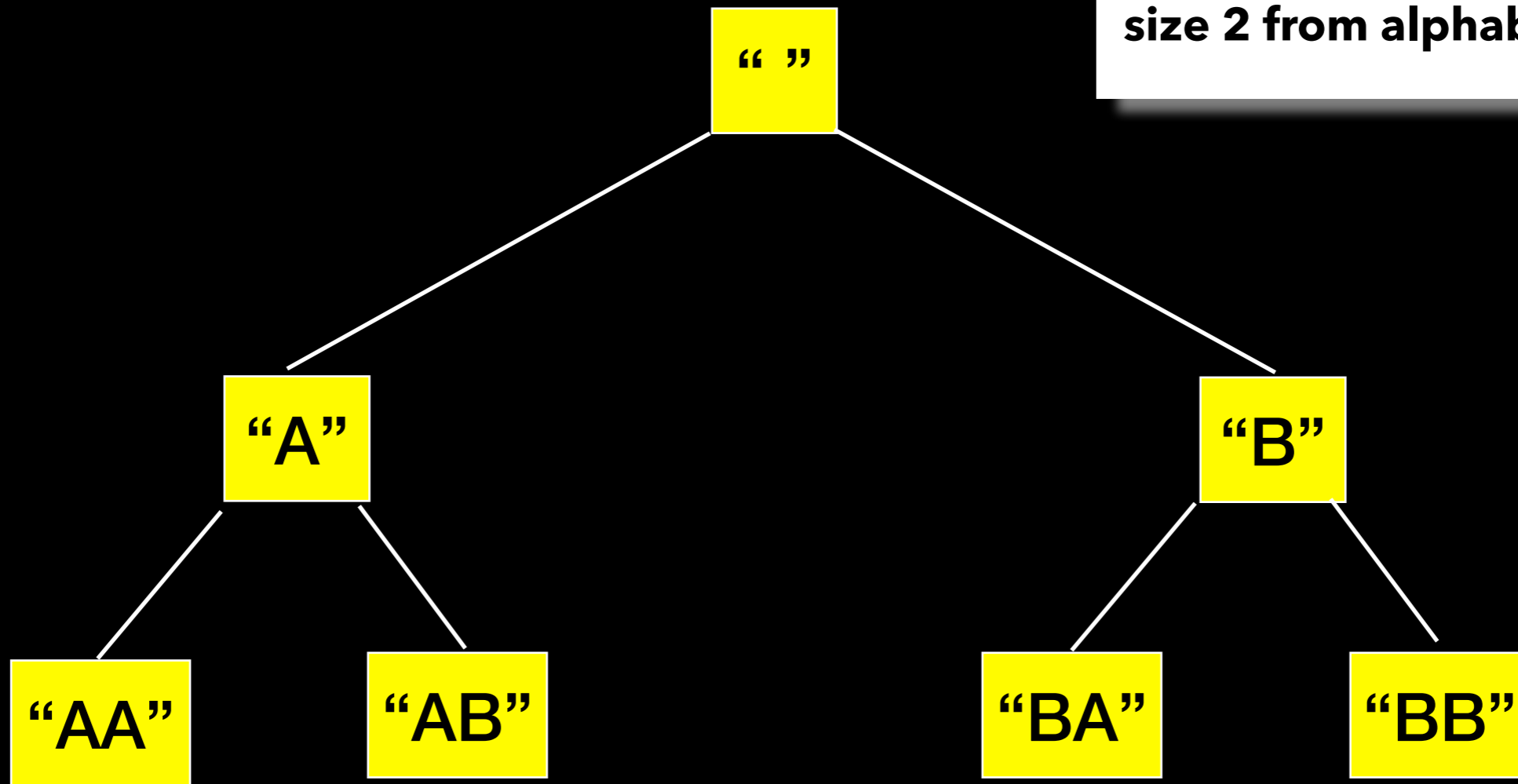


“BB“



{ "", "A", "B", "AA", "AB", "BA", "BB" }

Generate all substrings of size 2 from alphabet {'A', 'B'}



Breadth-First Search

Applications

Find shortest path in graph

GPS navigation systems

Crawlers in search engines

...

Generally good when looking for the "shortest" or "best" way to do something => lists things in increasing order of "size" stopping at the "shortest" solution

Size of Substring

```
findAllSubstrings(int n)
{
    put empty string on the queue


    while(queue is not empty){
        let current_string = dequeue and add to result
        if(size of current_string < n){
            for(each character ch)//every character in alphabet
                append ch to current_string and enqueue it
        }
    }
    return result;
}
```

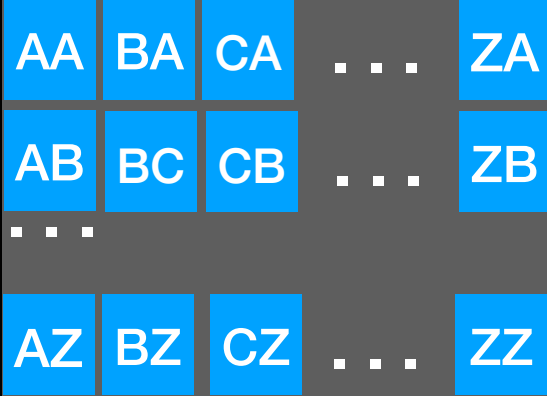
Analysis

Finding all substrings (with repetition) of size **up to n**

Assume **alphabet (A, B, ..., Z)** of size 26

The empty string = $1 = 26^0$ 

All strings of size 1 = 26^1 

All strings of size 2 = 26^2 

...

All strings of size n = 26^n

With repetition: I have 26 options for each of the n characters

Size of Substring

```
findAllSubstrings(int n)
{
    put empty string on the queue

    while(queue is not empty){
        let current_string = dequeue and add to result
        if(size of current_string < n){
            for(each character ch)//every character in alphabet
                append ch to current_string and enqueue it
        }
    }
    return result;
}
```

Analyze the worst-case time complexity of this algorithm assuming alphabet of size 26 and up to strings of length n

$T(n) = ?$

$O(?)$

Will stop when all strings have been removed from queue

```
findAllSubstrings(int n)
{
    put empty string on the queue
    while(queue is not empty){
        let current_string = dequeue and add to result
        if(size of current_string < n){
            for(each character ch)//every character in alphabet
                append ch to current_string and enqueue it
        }
    }
    return result;
}
```

Will stop when all strings have been removed from queue

Removes 1 string from the queue

Adds 26 strings to the queue

```
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        }
    }
    return result;
}
```

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    put empty string on the queue
    while(queue is not empty){
        let current_string = dequeue and add to result
        if(size of current_string < n){
            for(each character ch)//every character in alphabet
                append ch to current_string and enqueue it
        }
    }
    return result;
}
```

Loop until queue is empty and dequeue only 1 each time.

So the question becomes:

How many strings are enqueued in total?

Will stop when all strings have been removed from queue

Removes 1 string from the queue

Adds 26 strings to the queue

```
findAllSubstrings(int n)
{
    put empty string on the queue
    while(queue is not empty){
        let current_string = dequeue and add to result
        if(size of current_string < n){
            for(each character ch)//every character in alphabet
                append ch to current_string and enqueue it
        }
    }
    return result;
}
```

$$T(n) = 26^0 + 26^1 + 26^2 + \dots + 26^n$$

Will stop when all strings have been removed from queue

Removes 1 string from the queue

Adds 26 strings to the queue

```
findAllSubstrings(int n)
{
    put empty string on the queue
    while(queue is not empty){
        let current_string = dequeue and add to result
        if(size of current_string < n){
            for(each character ch)//every character in alphabet
                append ch to current_string and enqueue it
        }
    }
    return result;
}
```

$$T(n) = 26^0 + 26^1 + 26^2 + \dots + 26^n$$

Will stop when all strings have been removed from queue

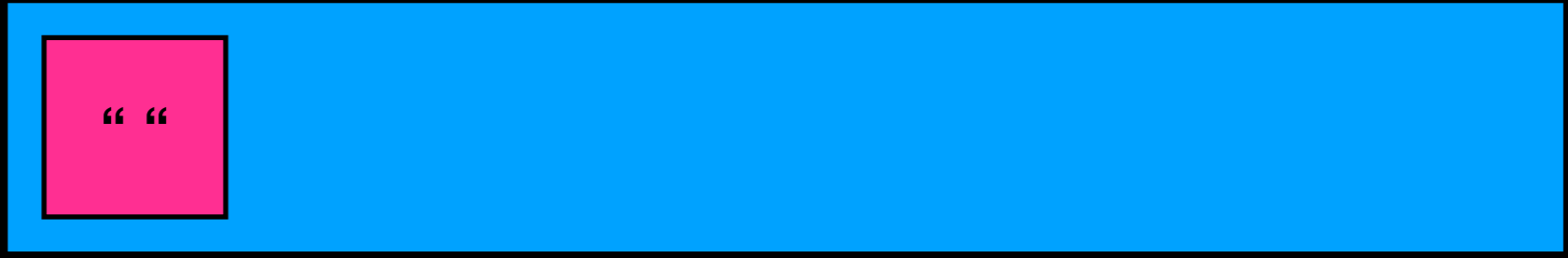
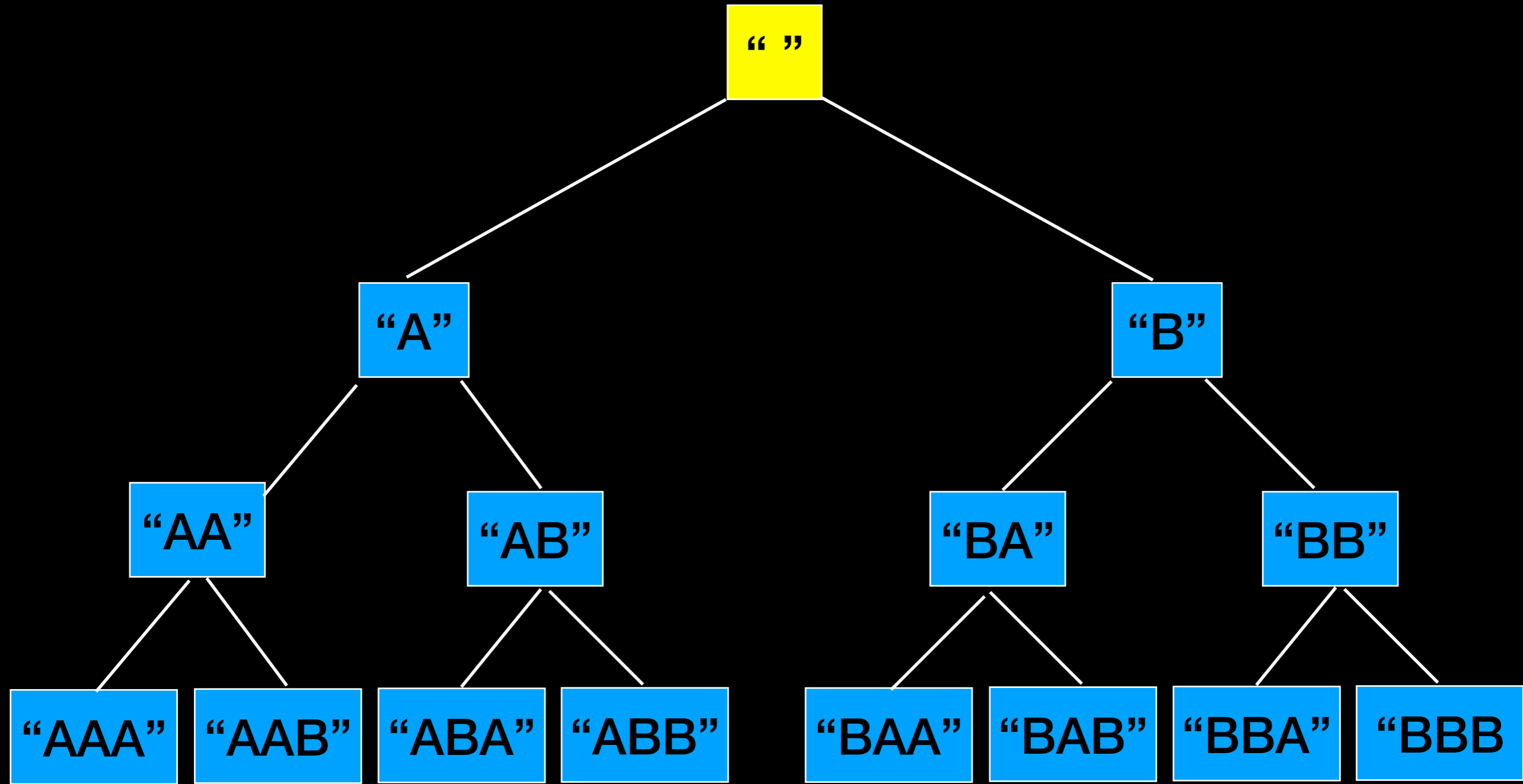
Removes 1 string from the queue

Adds 26 strings to the queue

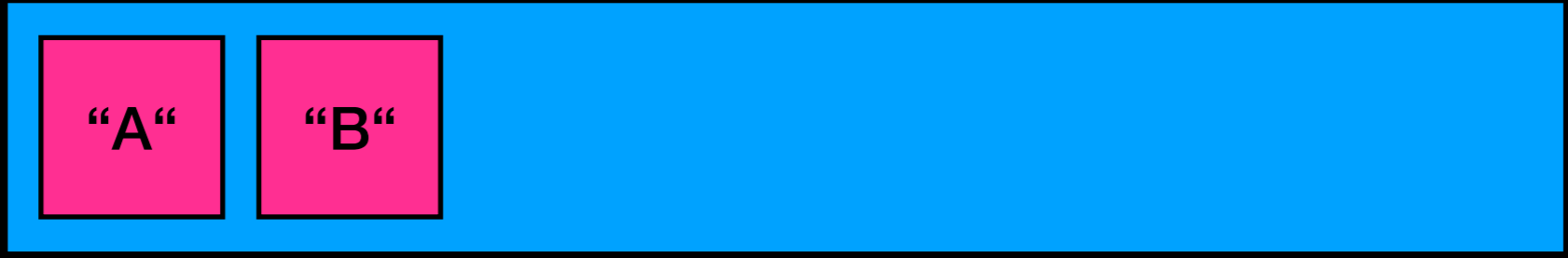
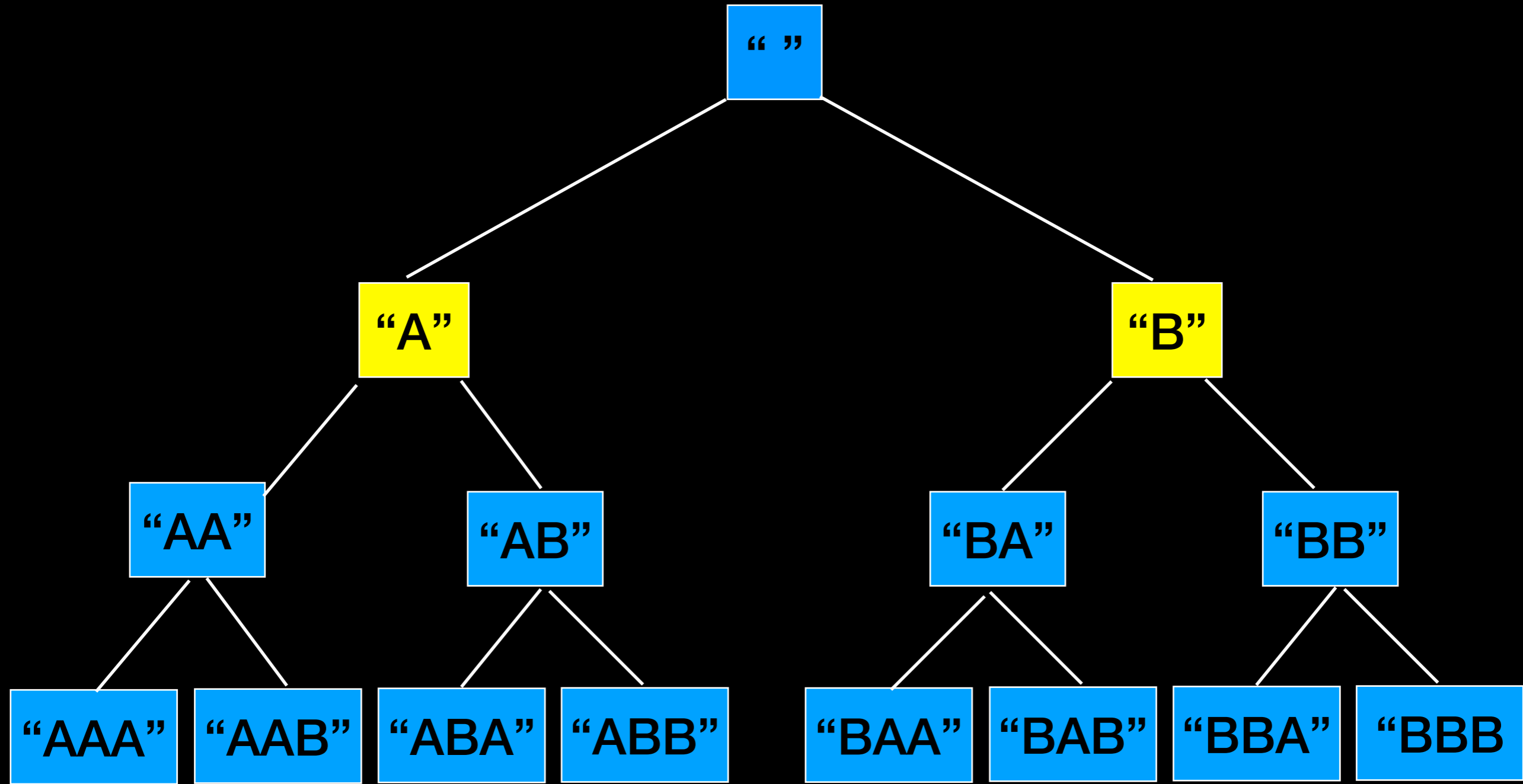
```
findAllSubstrings(int n)
{
    put empty string on the queue
    while(queue is not empty){
        let current_string = dequeue and add to result
        if(size of current_string < n){
            for(each character ch)//every character in alphabet
                append ch to current_string and enqueue it
        }
    }
    return result;
}
```

$O(26^n)$

Let $n = 3$, alphabet still $\{ 'A', 'B' \}$

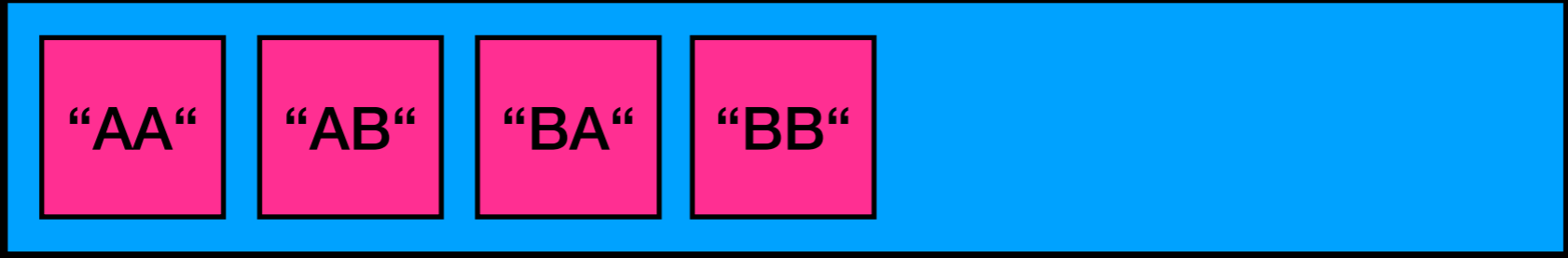
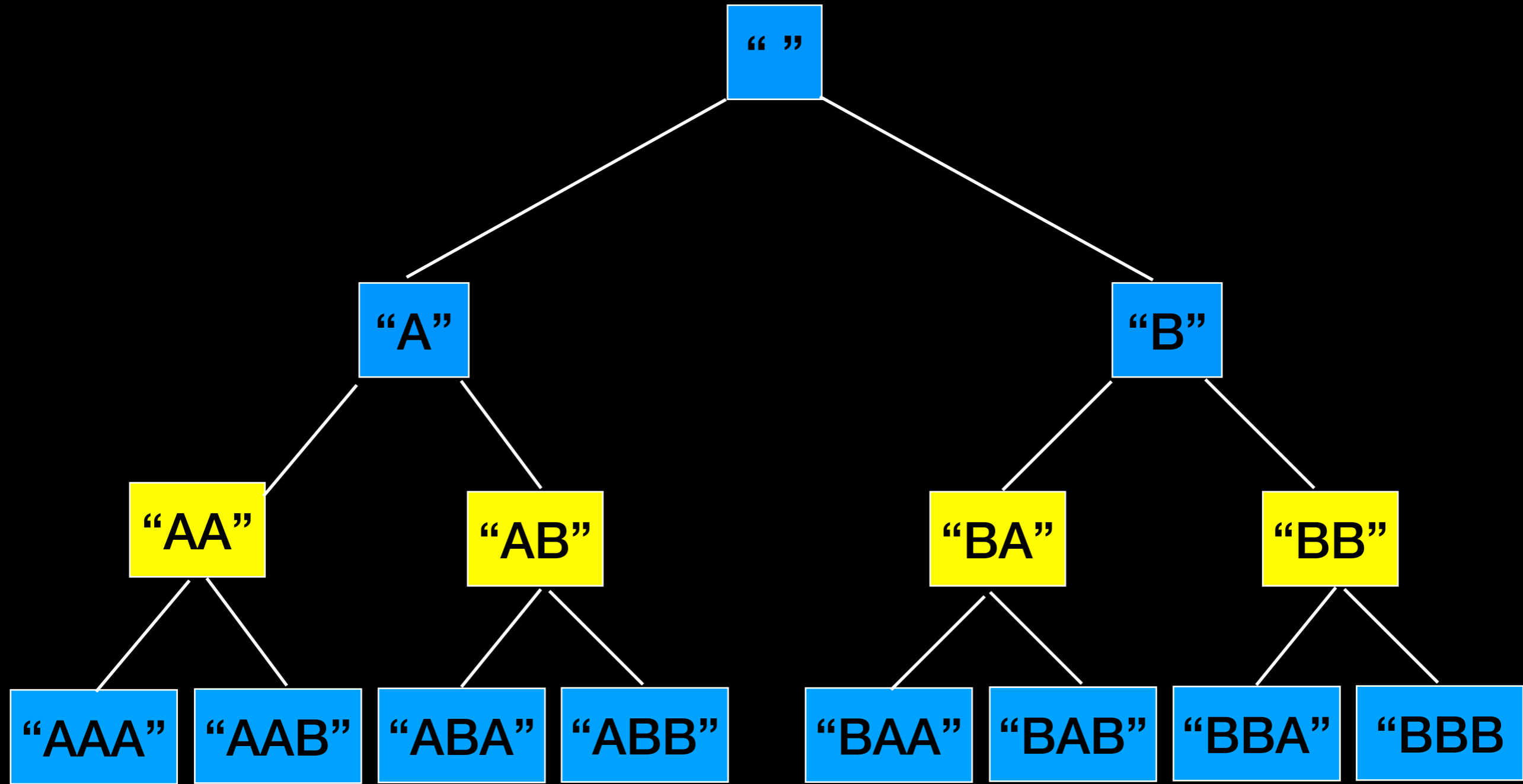


Let $n = 3$, alphabet still $\{ 'A', 'B' \}$



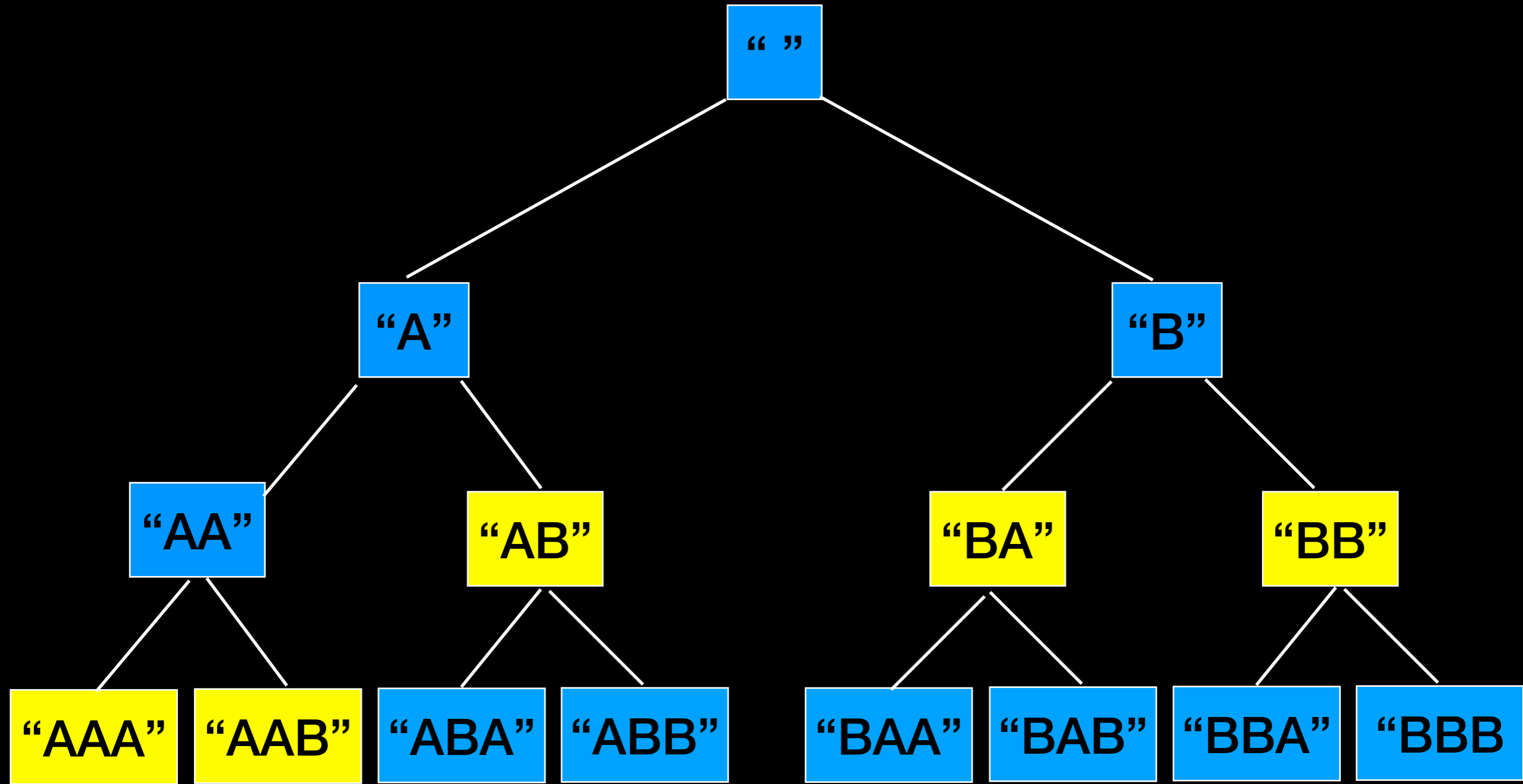
21

Let $n = 3$, alphabet still $\{ 'A', 'B' \}$



2^2

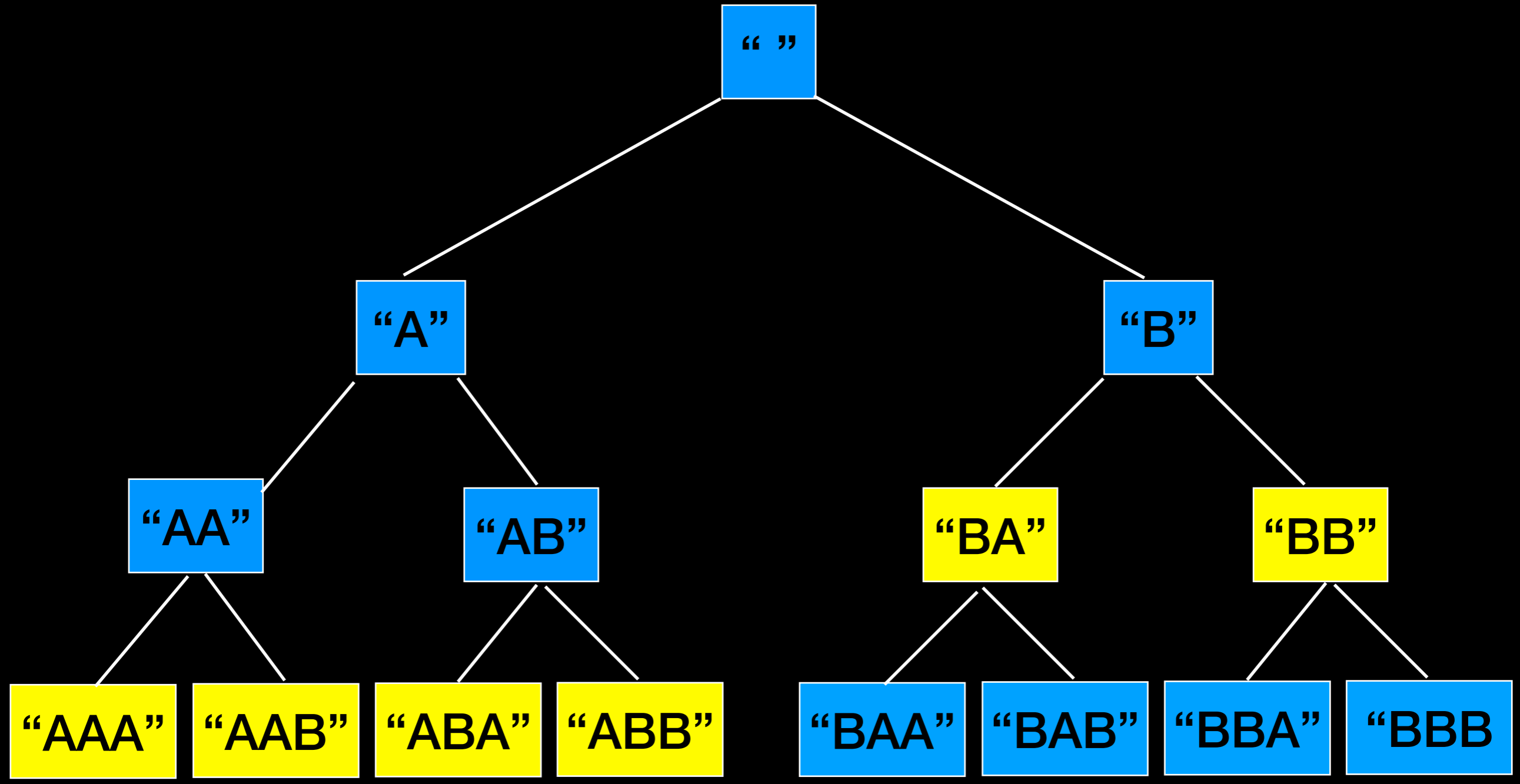
Let $n = 3$, alphabet still $\{ 'A', 'B' \}$



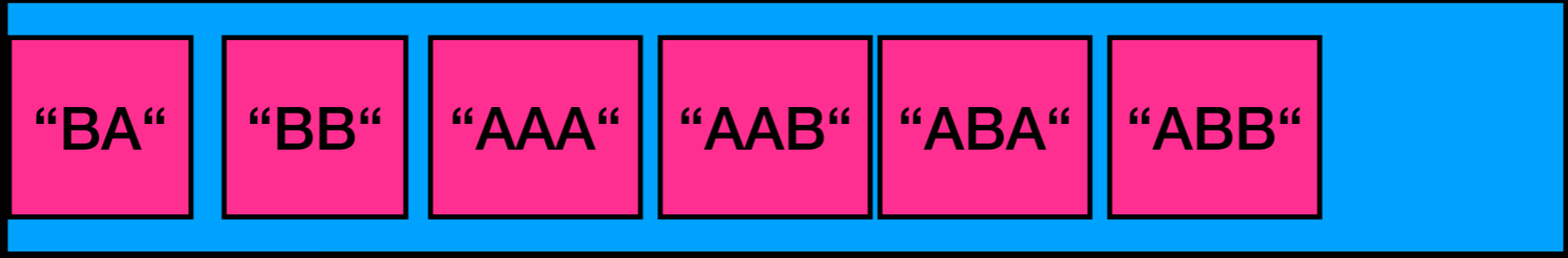
“AA”

“AB” “BA” “BB” “AAA” “AAB”

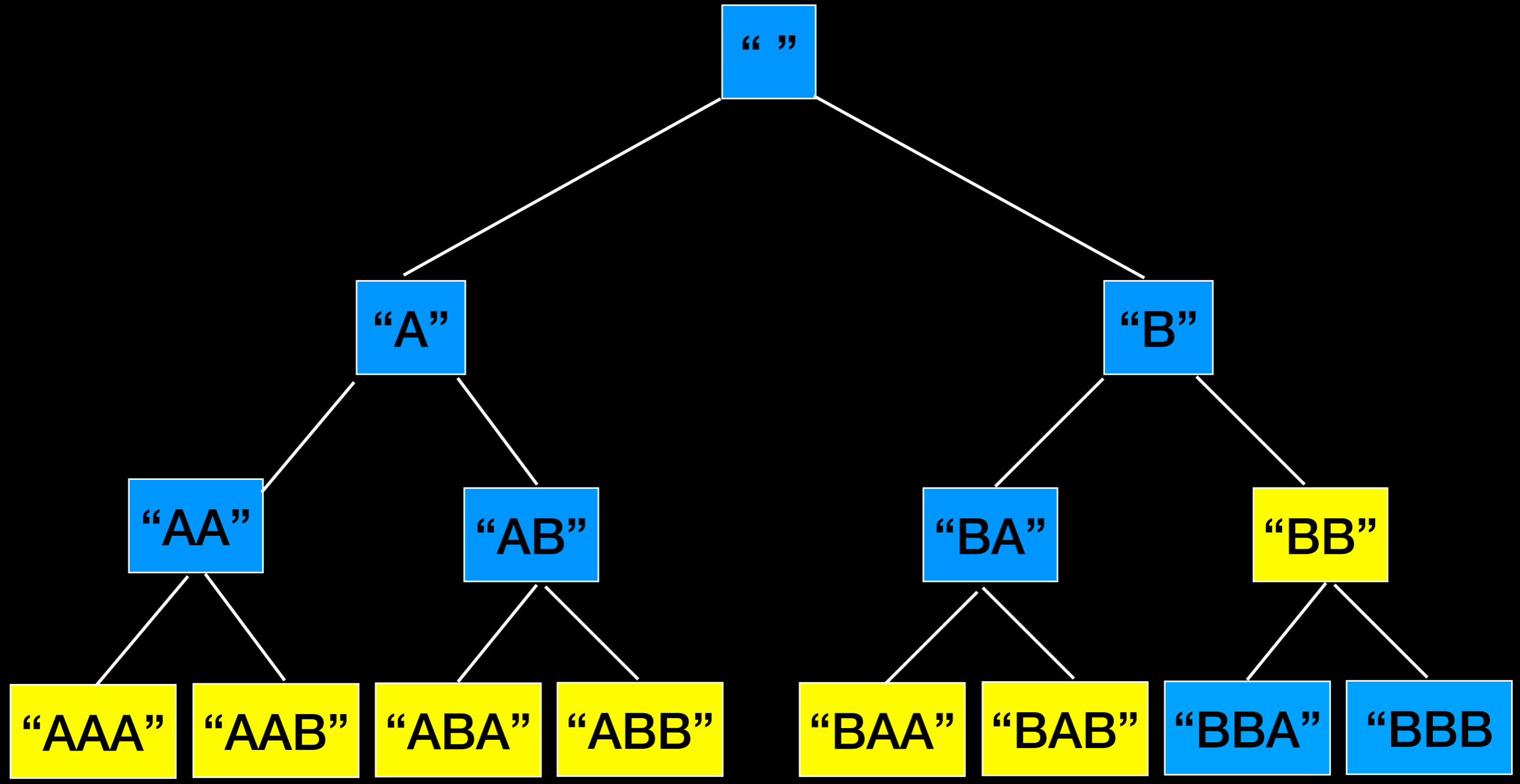
Let $n = 3$, alphabet still $\{ 'A', 'B' \}$



“AB“



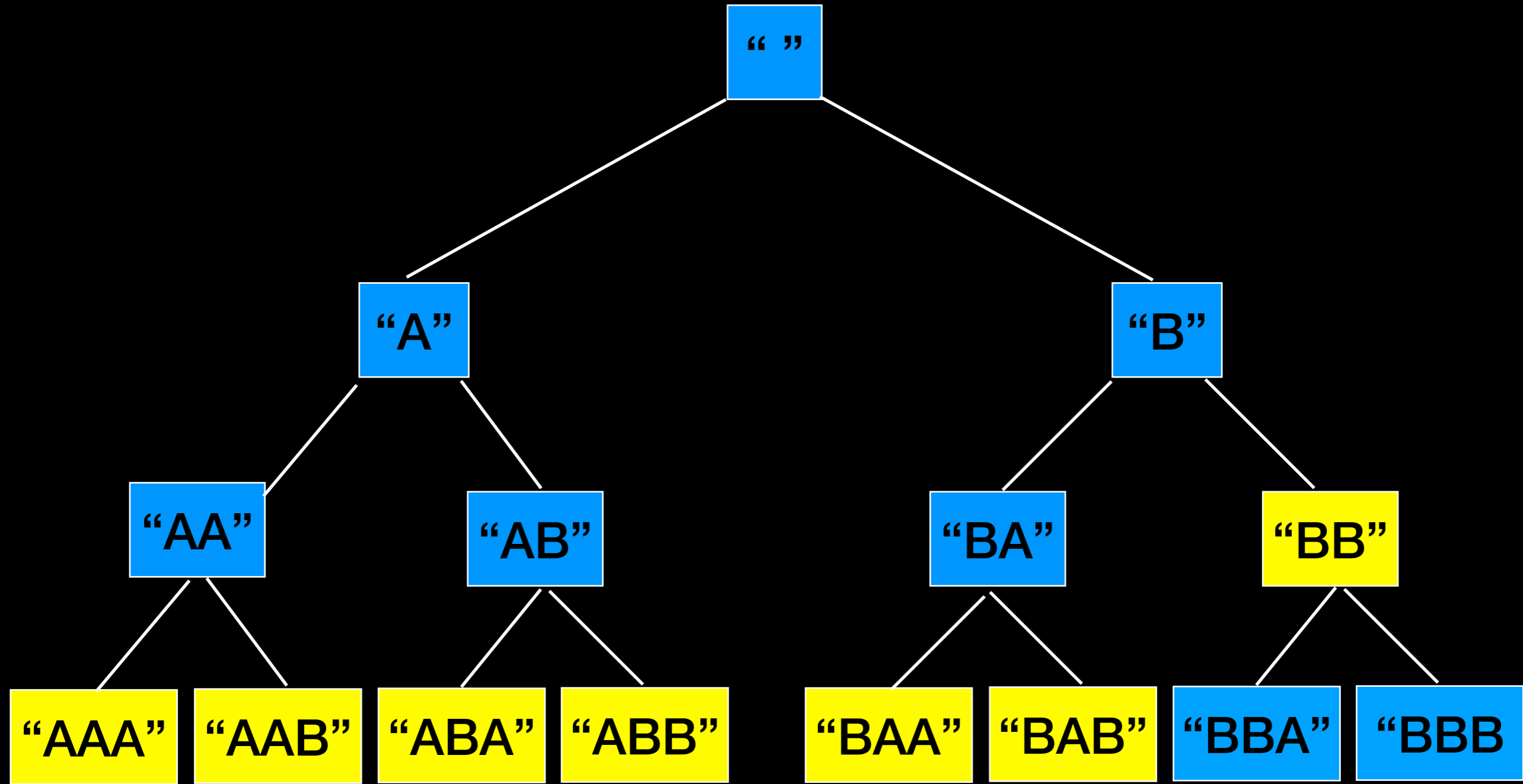
Let $n = 3$, alphabet still $\{ 'A', 'B' \}$



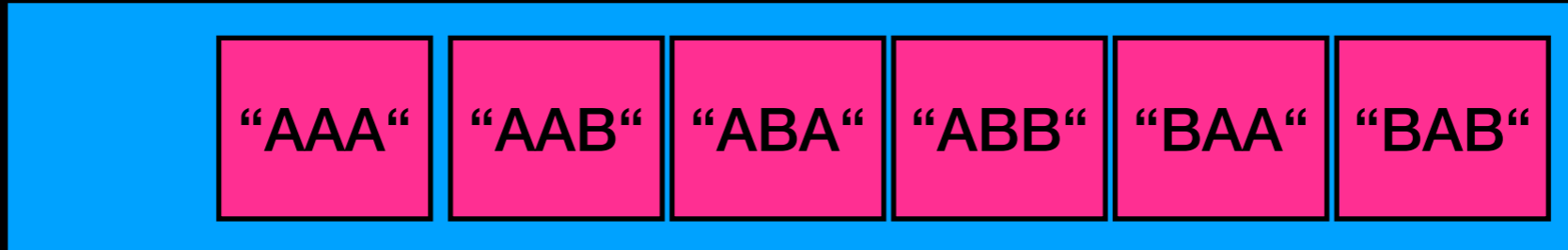
“BA“

- “BB“
- “AAA“
- “AAB“
- “ABA“
- “ABB“
- “BAA“
- “BAB“

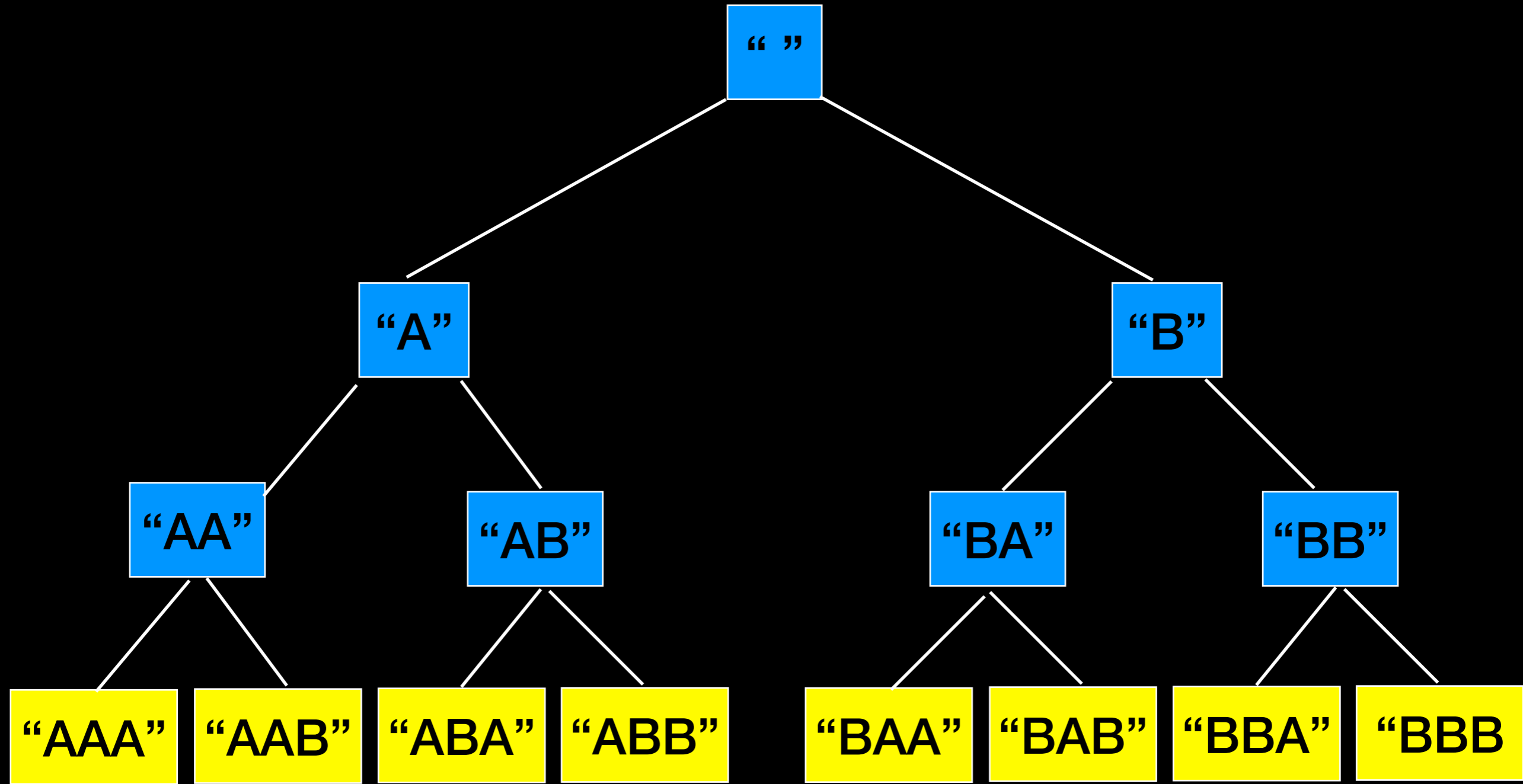
Let $n = 3$, alphabet still $\{ 'A', 'B' \}$



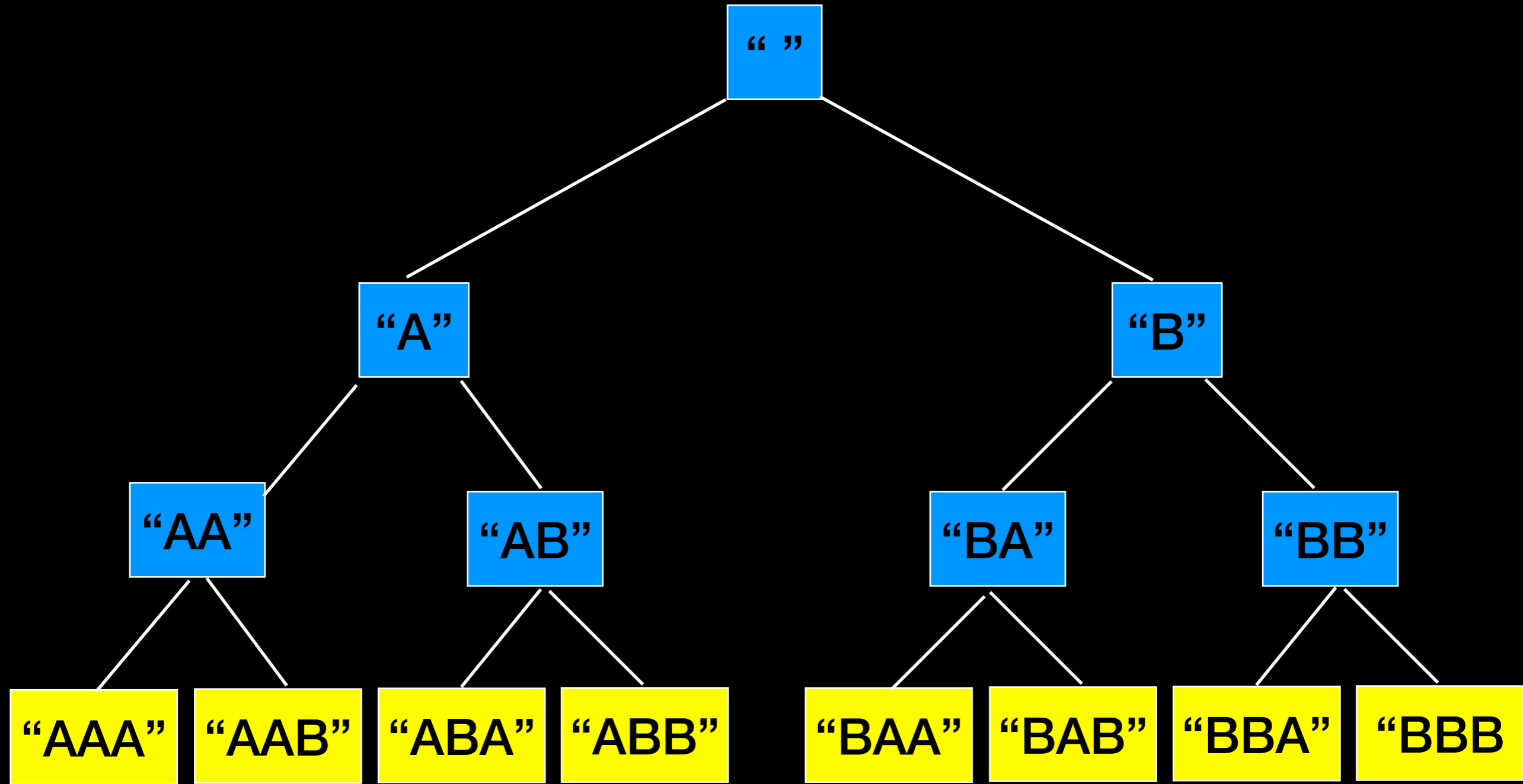
“BB”



Let $n = 3$, alphabet still $\{ 'A', 'B' \}$



Let $n = 3$, alphabet still $\{ 'A', 'B' \}$



- “AAA“
- “AAB“
- “ABA“
- “ABB“
- “BAA“
- “BAB“
- “BBA“
- “BBB“

2^3

Memory Usage

With alphabet {'A', 'B', ..., 'Z'}, at some point we end up with 26^n strings in memory

Size of string on my machine = 24 bytes

Running this algorithm for $n = 7$ ($\approx 193\text{GB}$) is the maximum that can be handled by a standard personal computer

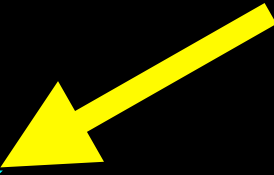
For $n = 8 \approx 5\text{TB}$



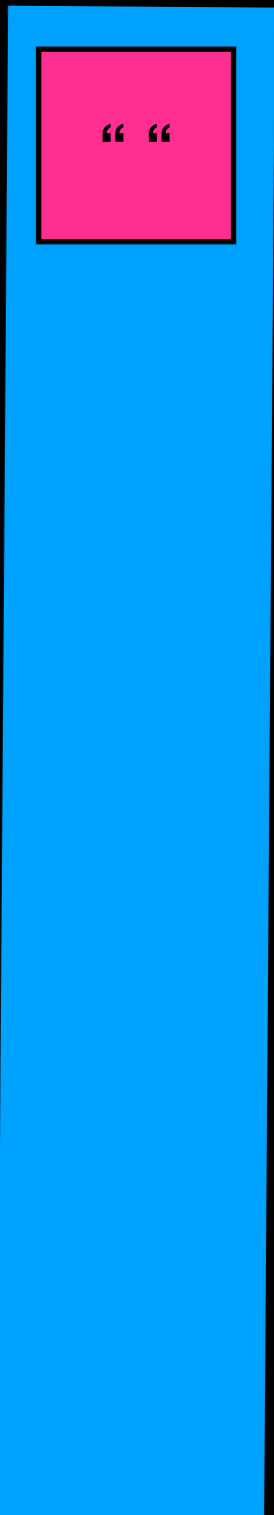
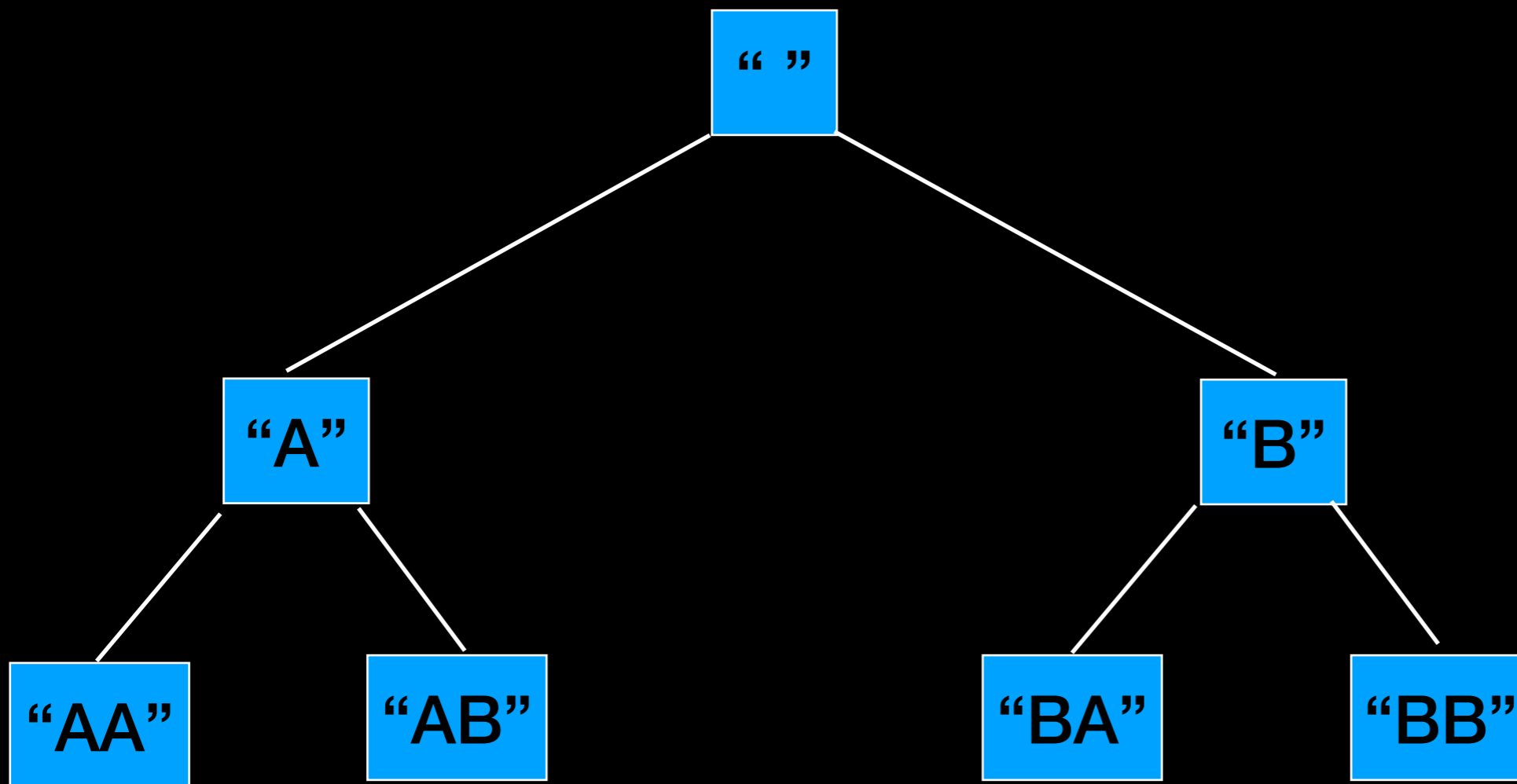
Massive
space
requirement

What if we use a stack?

```
findAllSubstrings(int n)
{
    push empty string on the stack
    while(stack is not empty){
        let current_string = pop and add to result
        if(size of current_string < n){
            for(each character ch)//every character in alphabet
                append ch to current_string and push it
        }
    }
    return result;
}
```

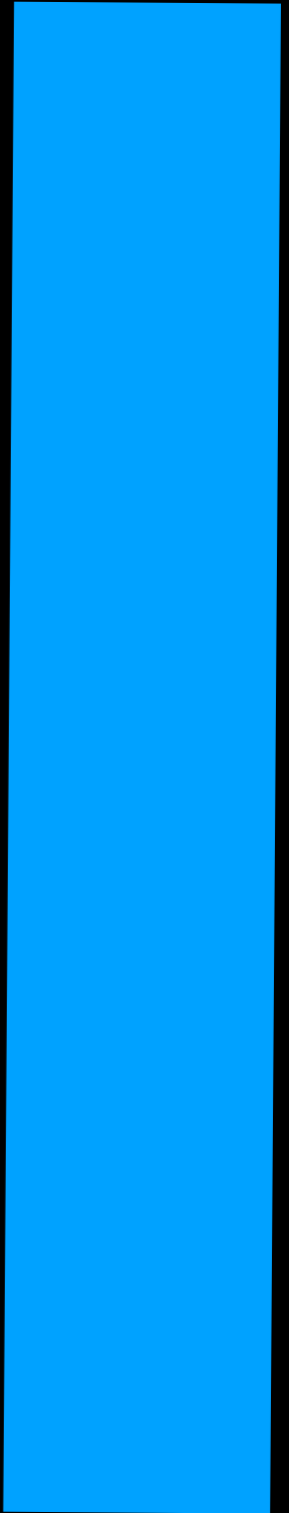
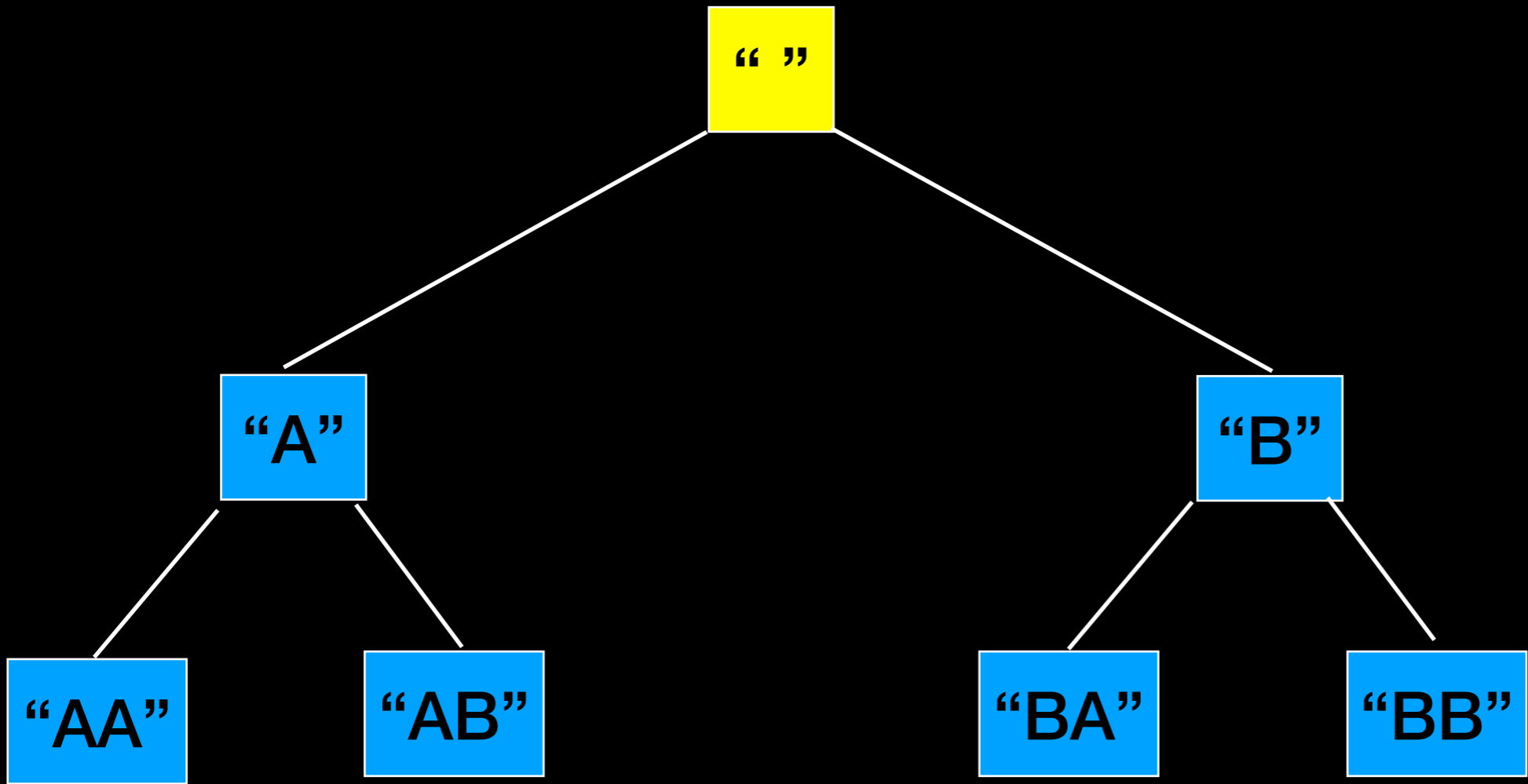


$O(26^n)$



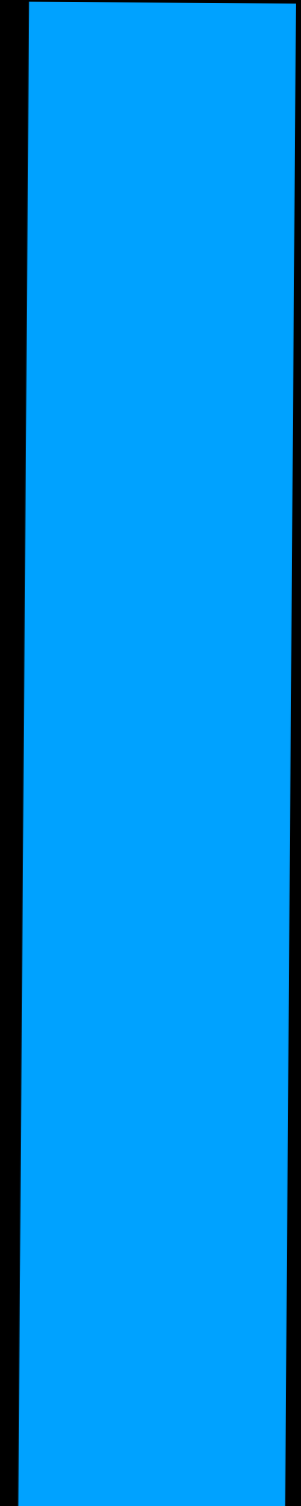
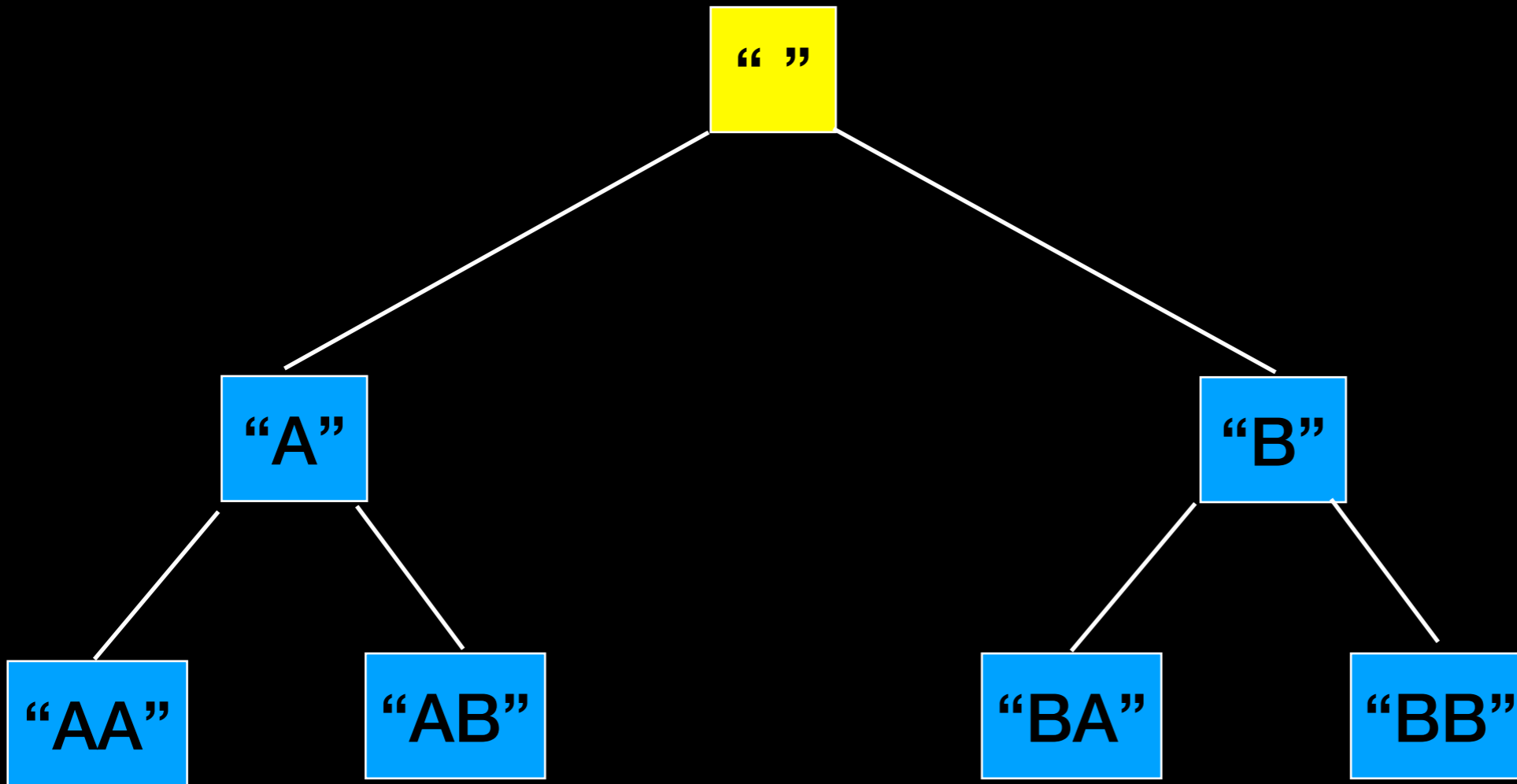
{ "" }

“ ”

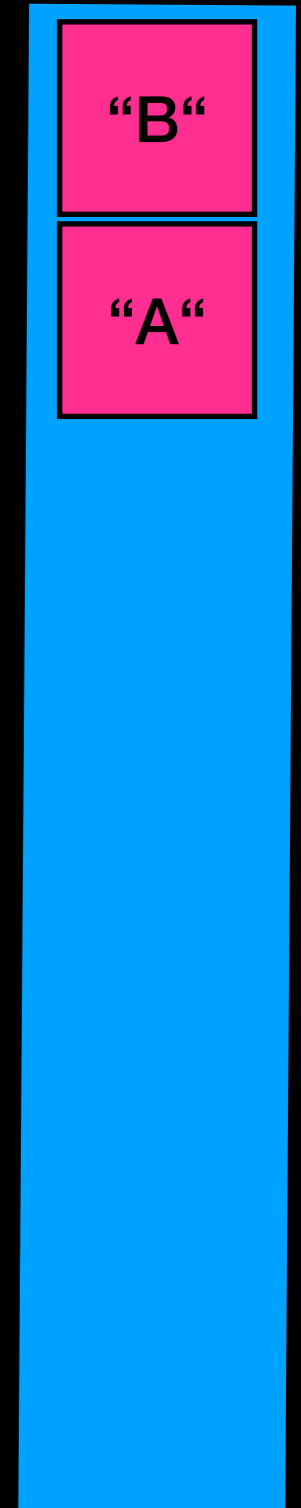
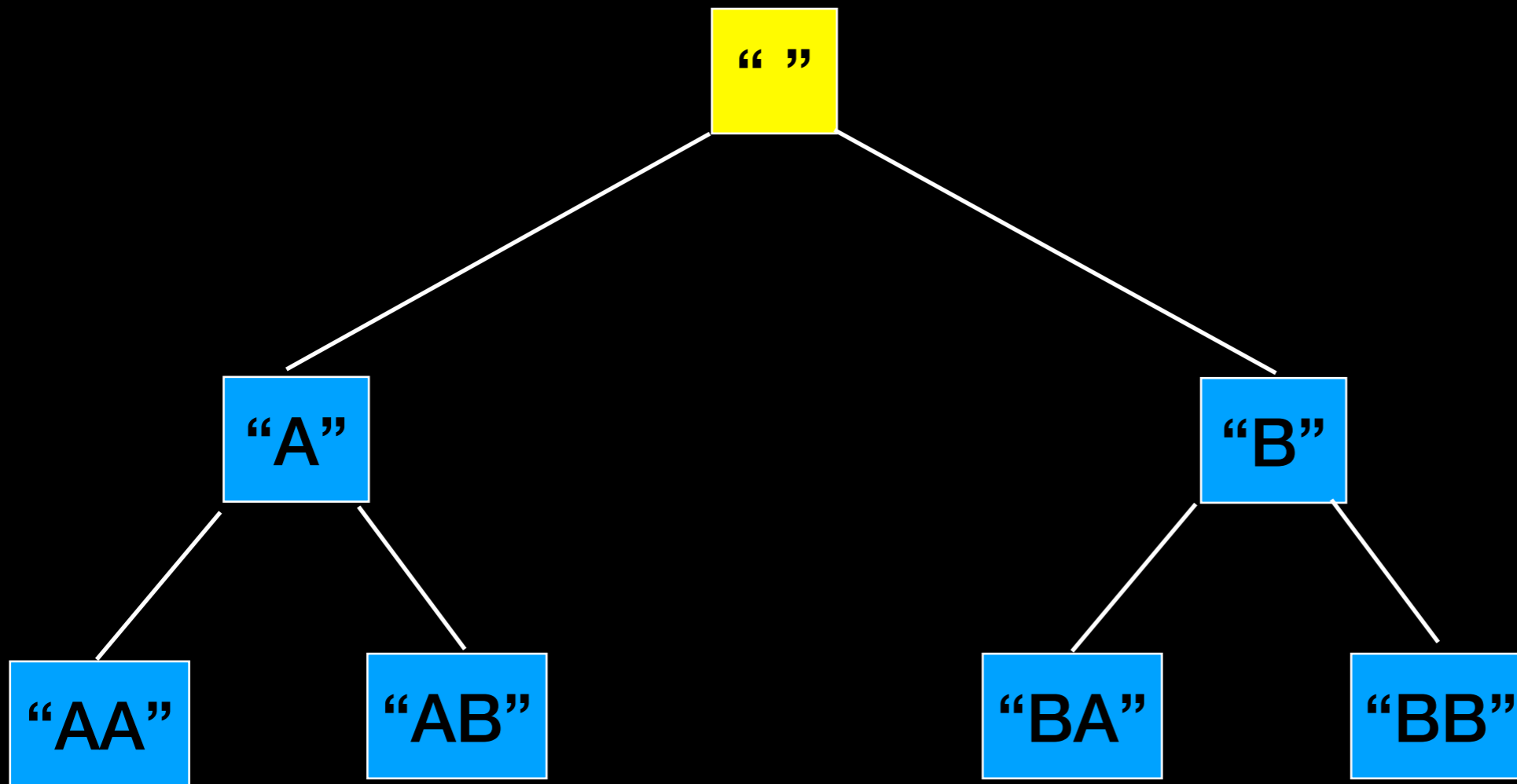


{ "" }

“ “ “A” “B”

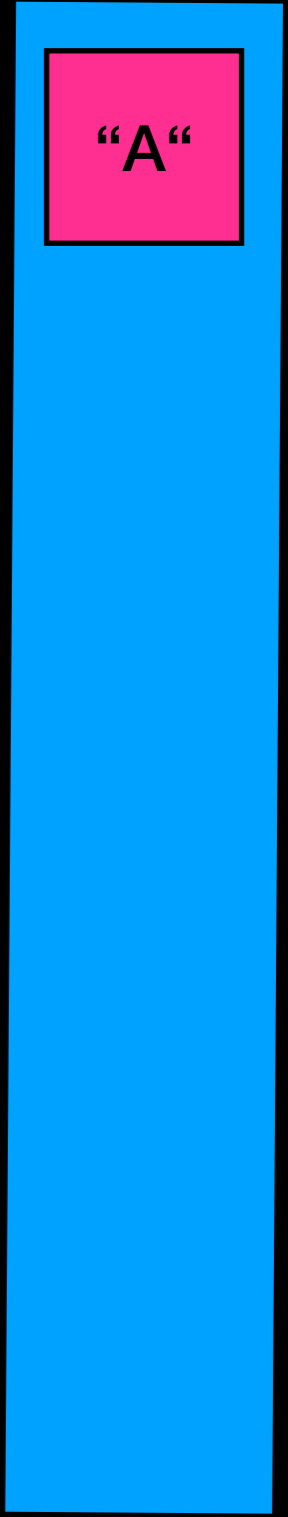
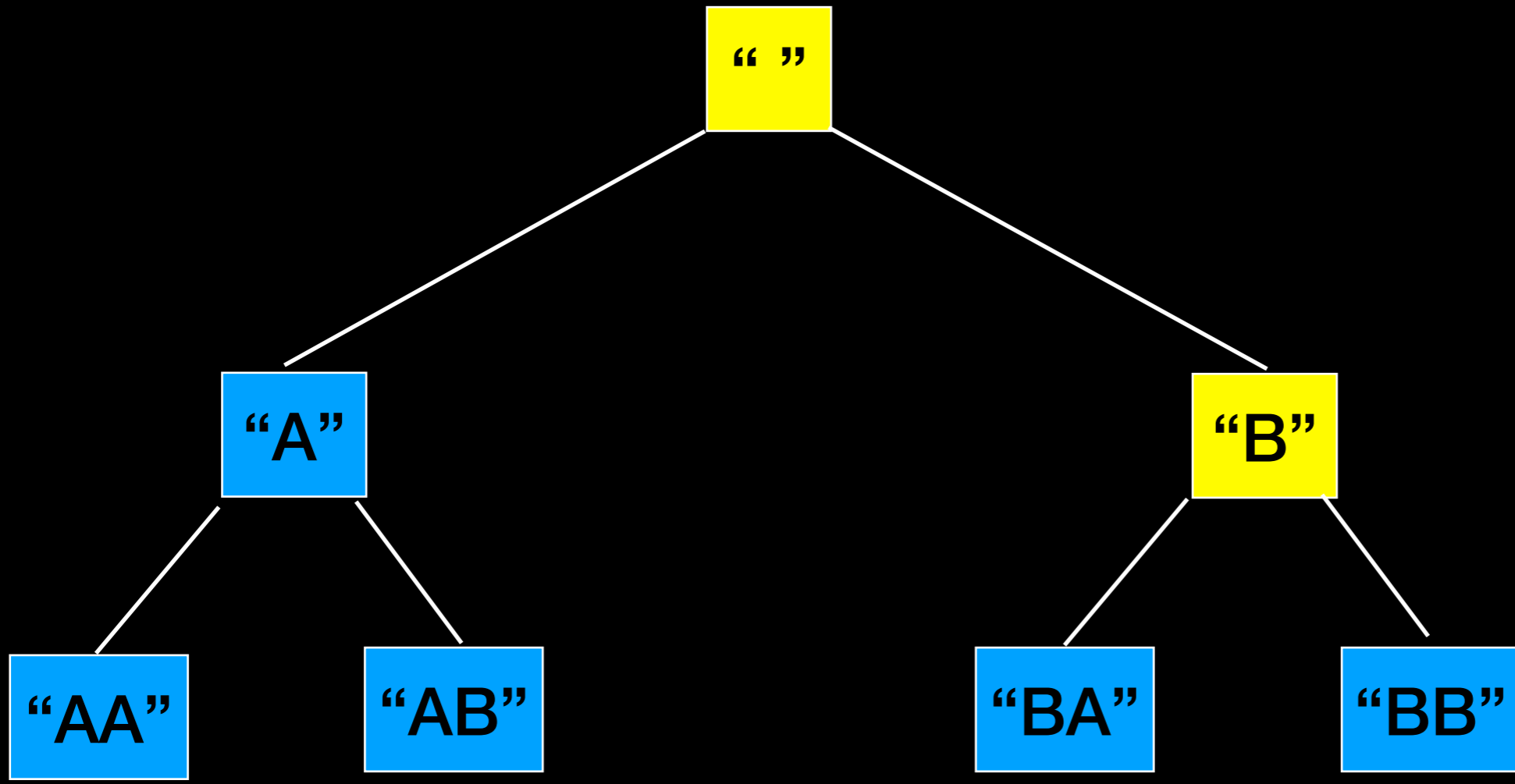


{ "" }



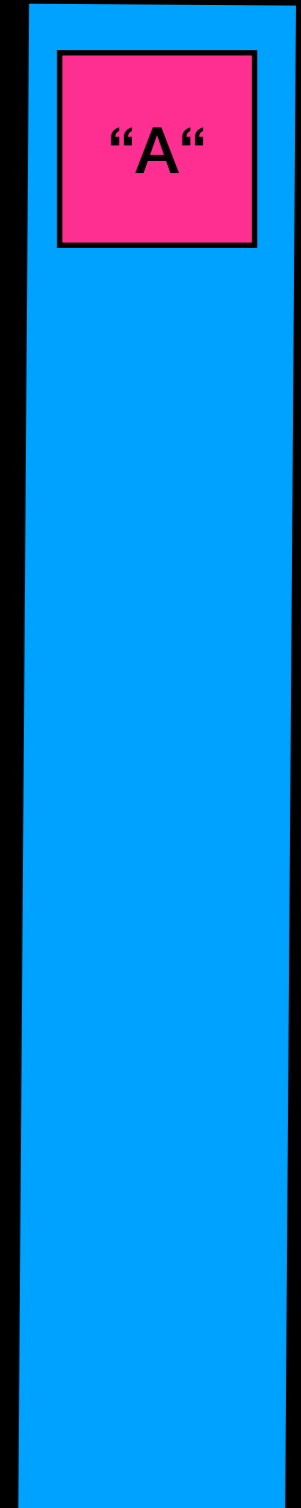
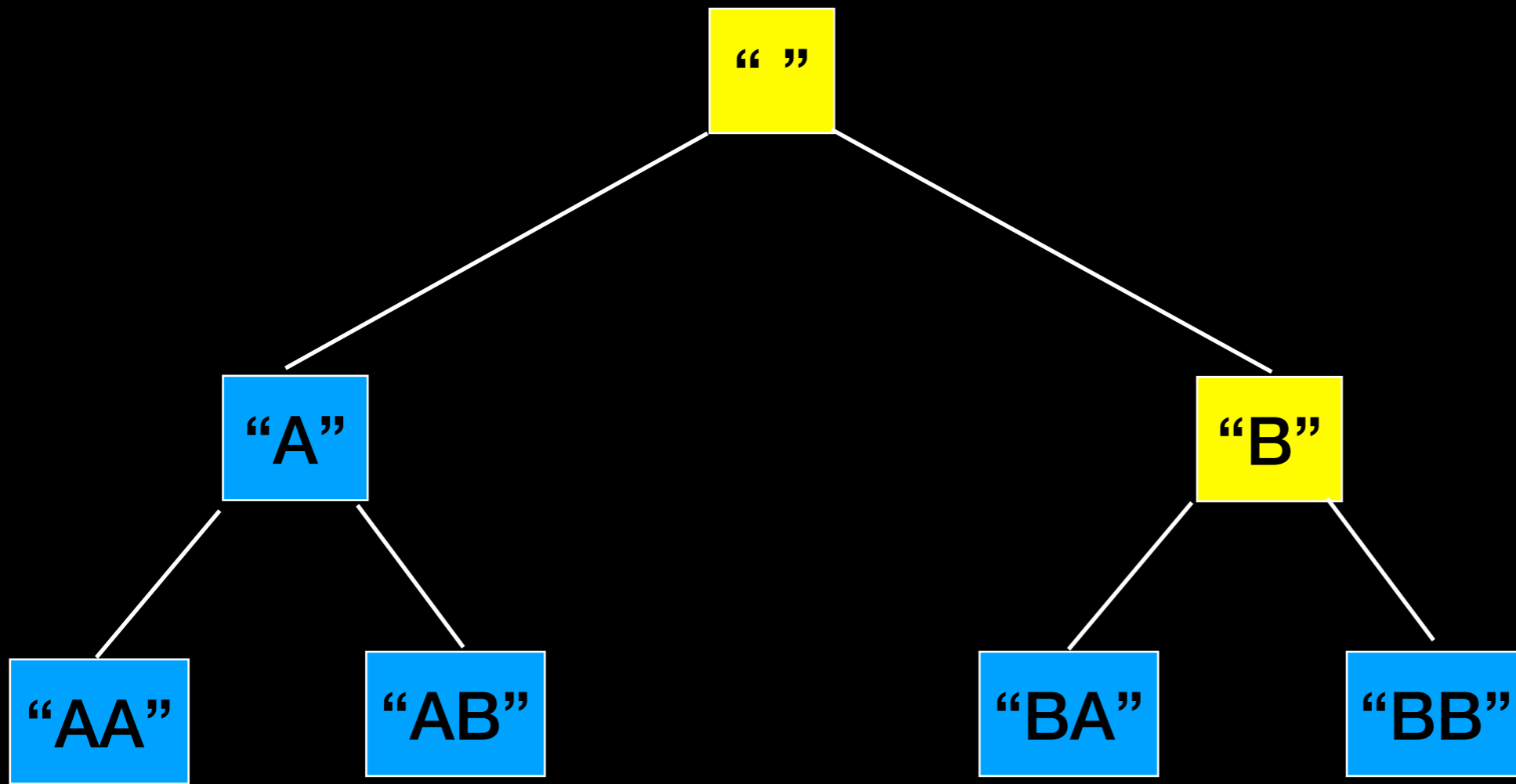
{ "" }

“B”

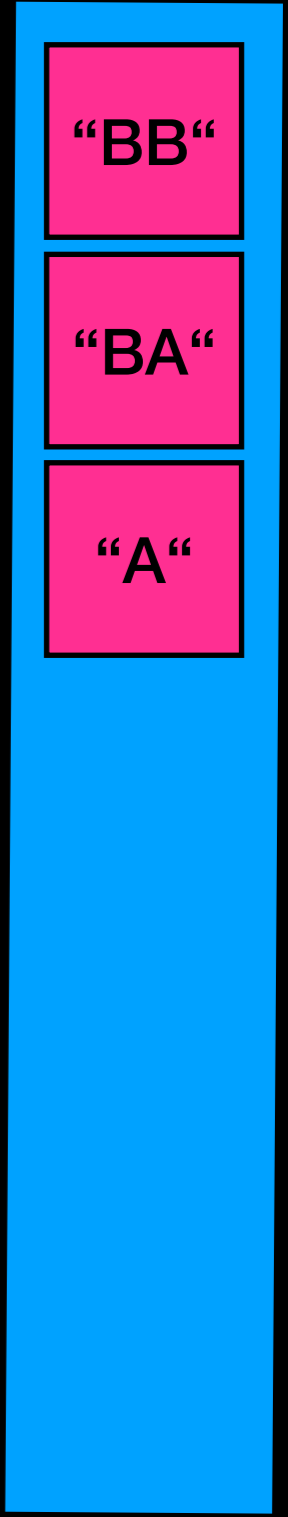
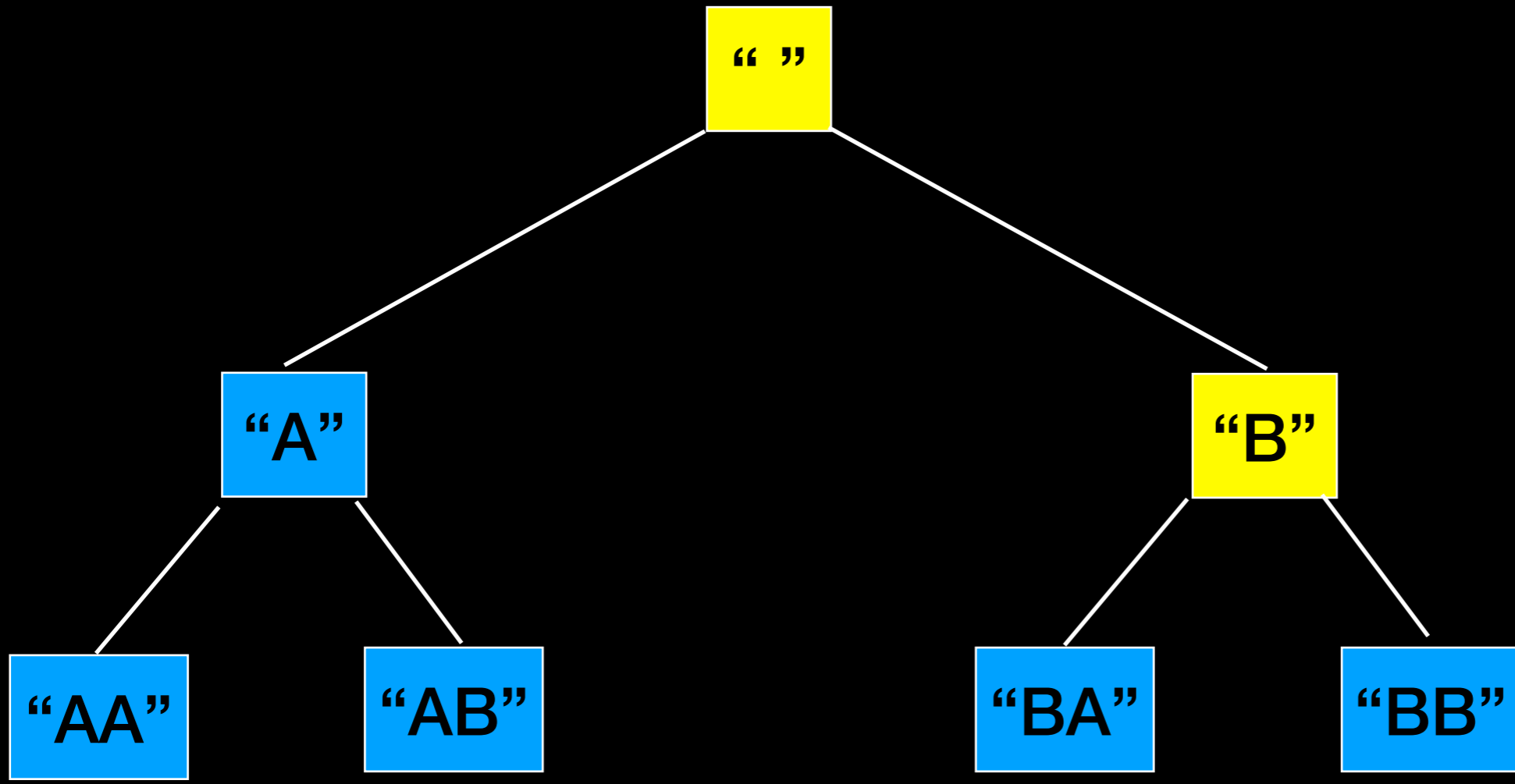


{ "", "B" }

"B" "BA" "BB"

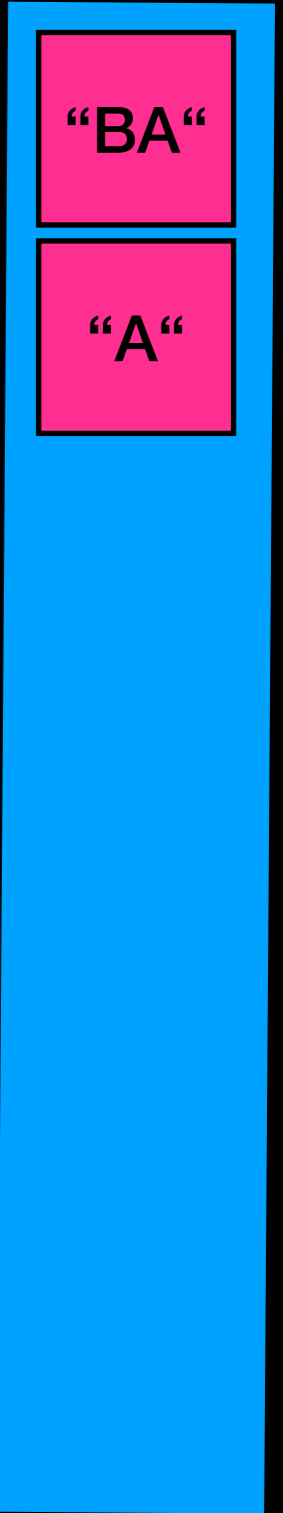
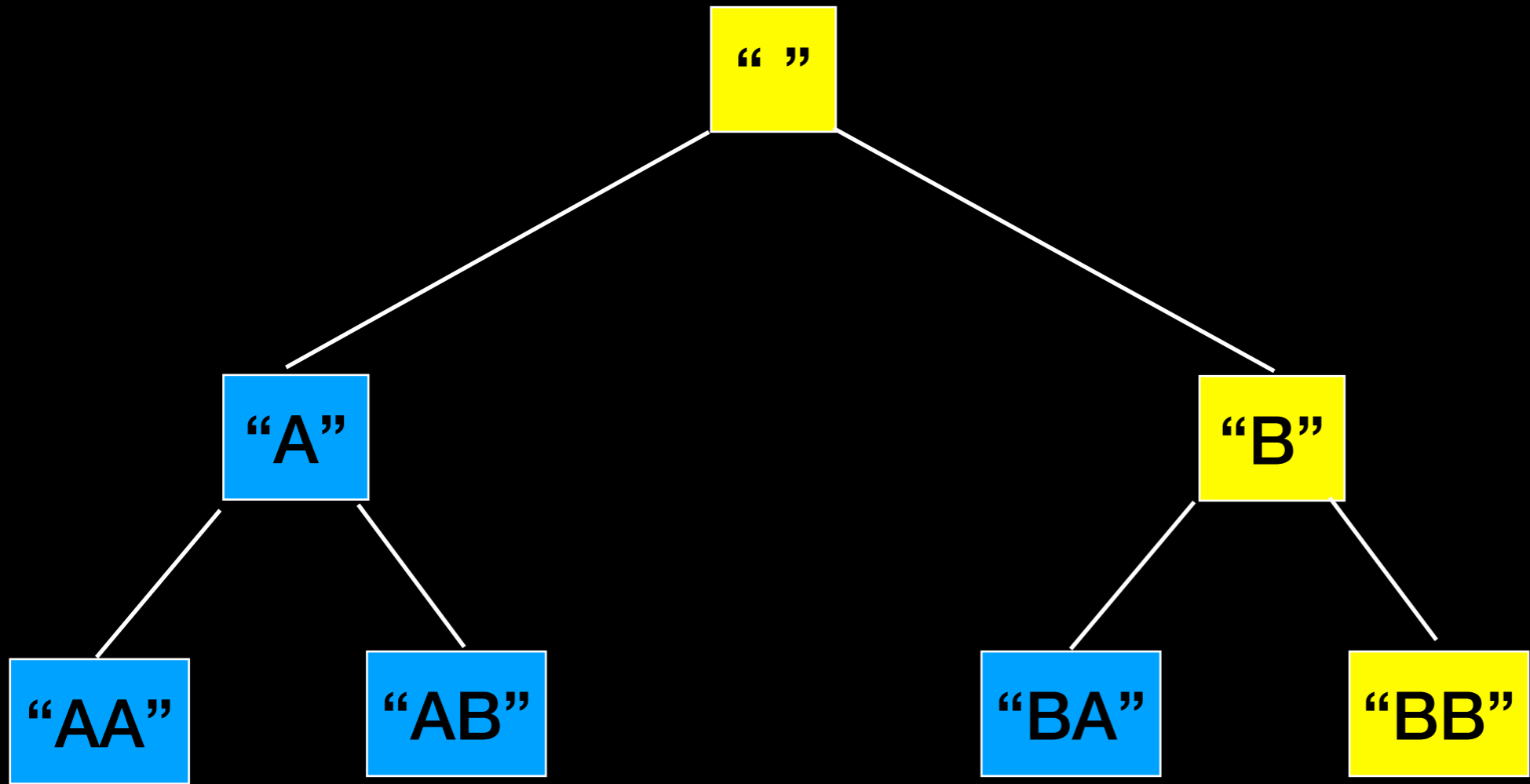


{ "", "B" }



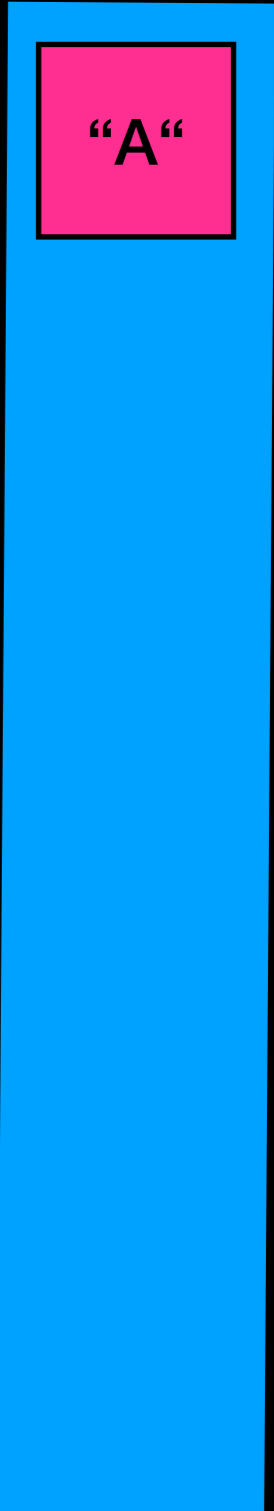
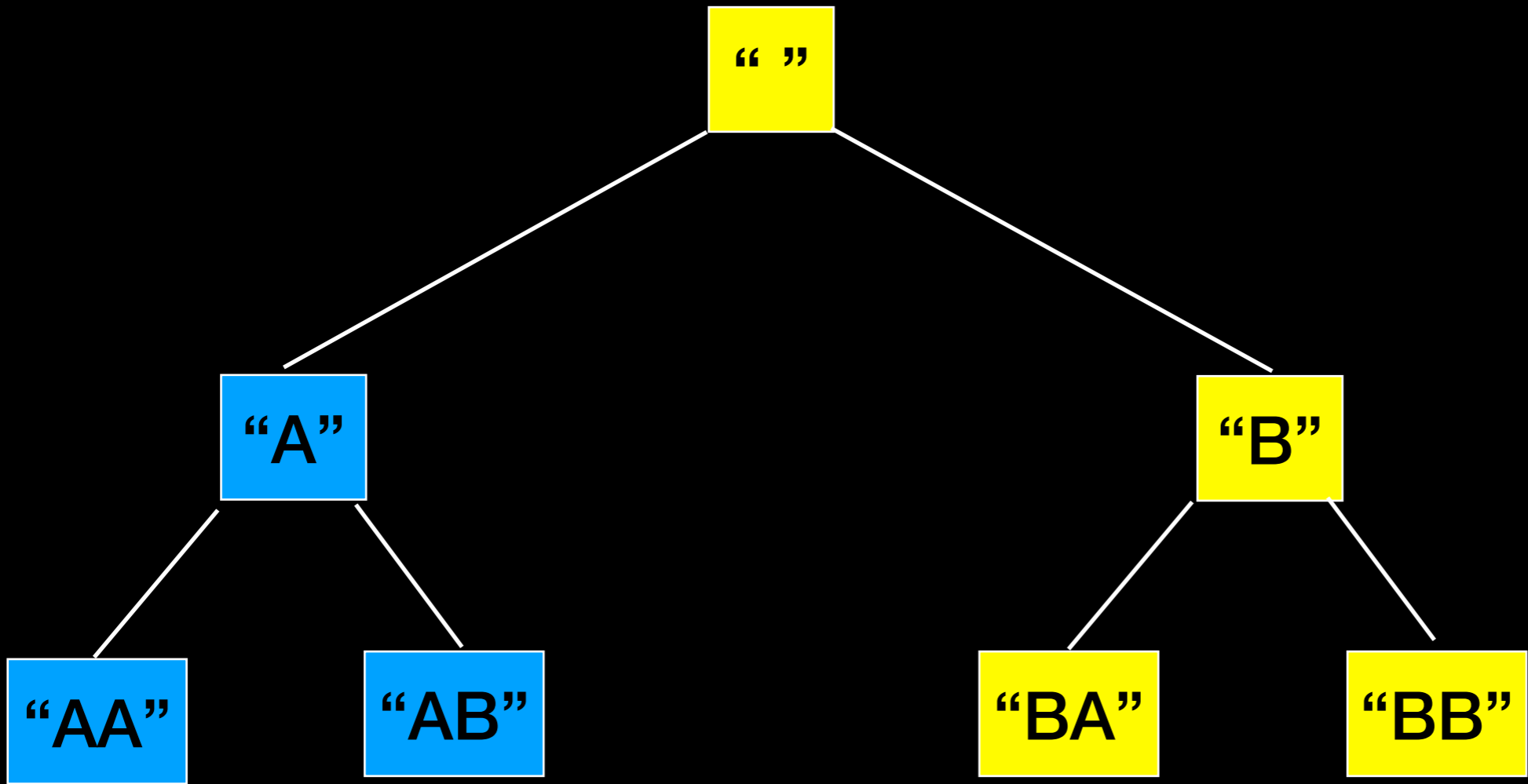
{ "", "B", "BB" }

"BB"



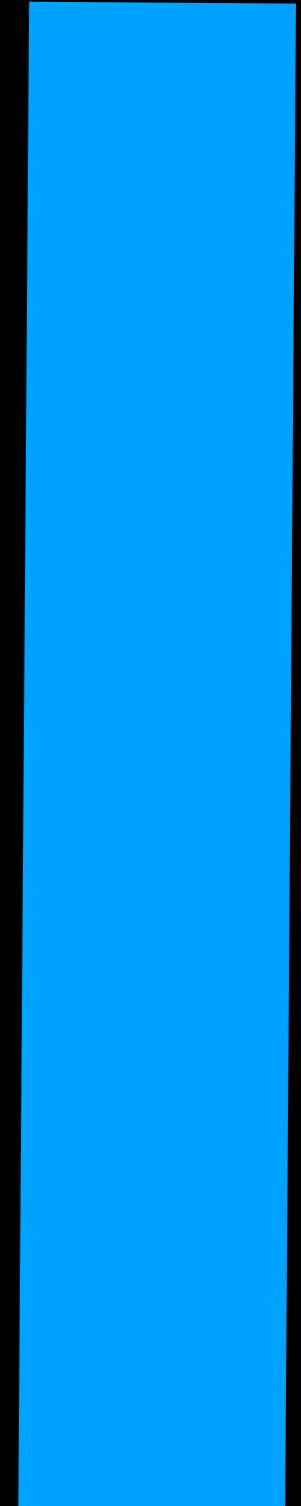
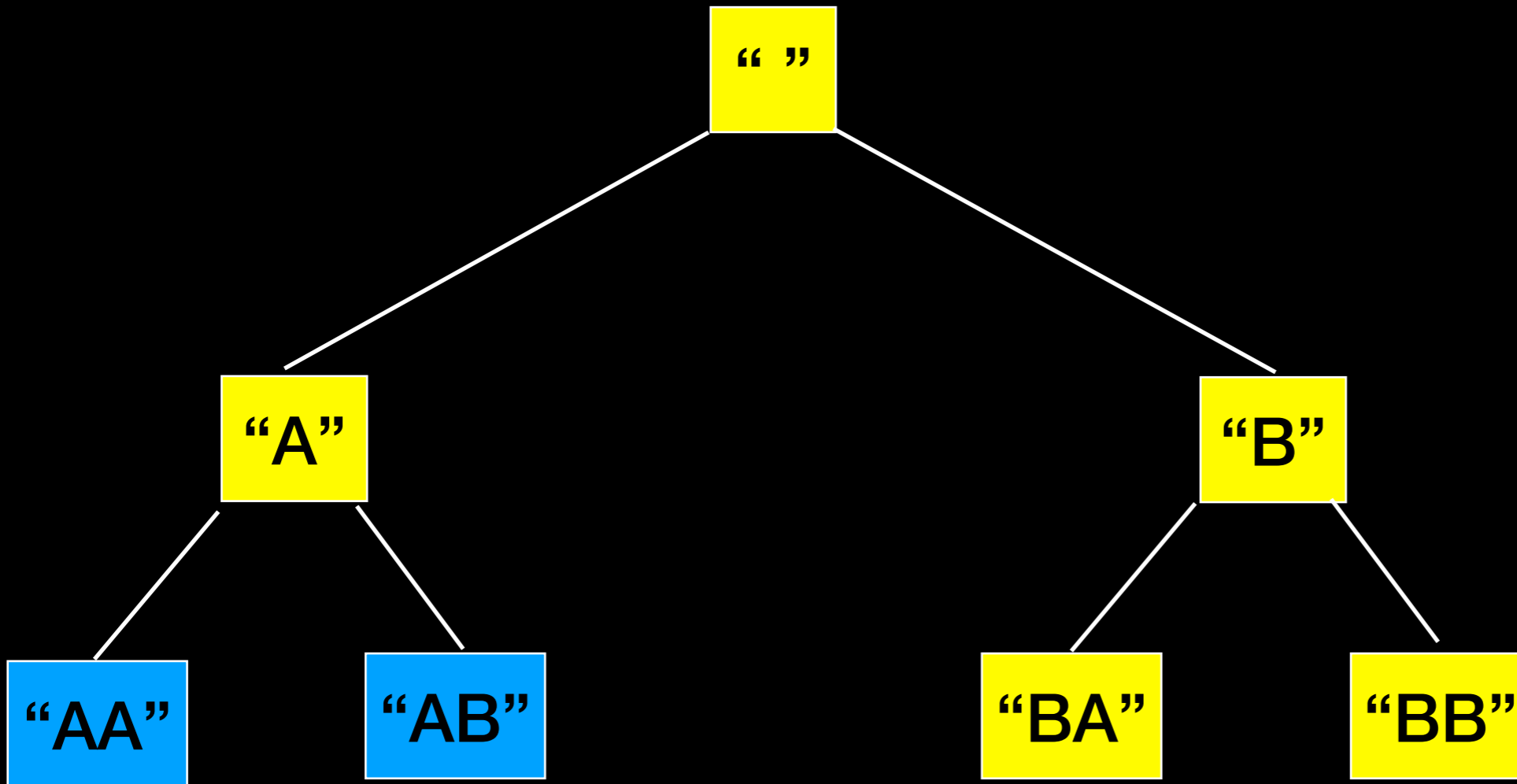
{ "", "B", "BB", "BA" }

"BA"



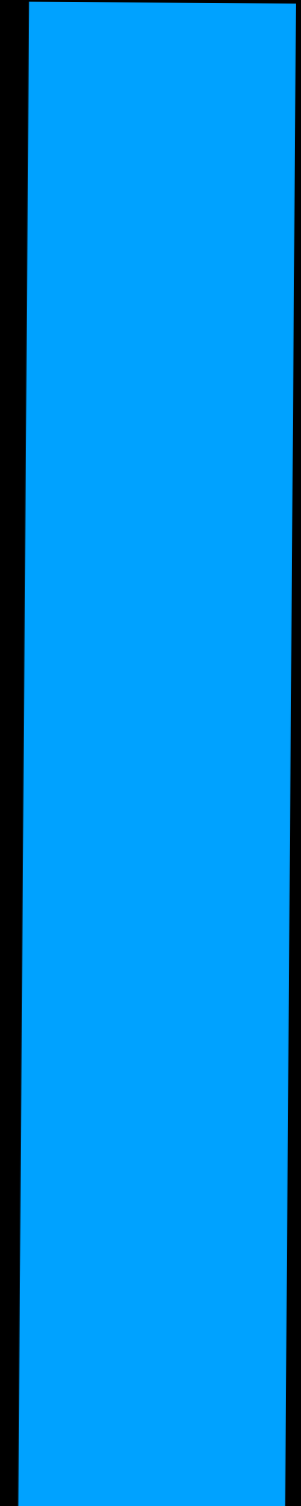
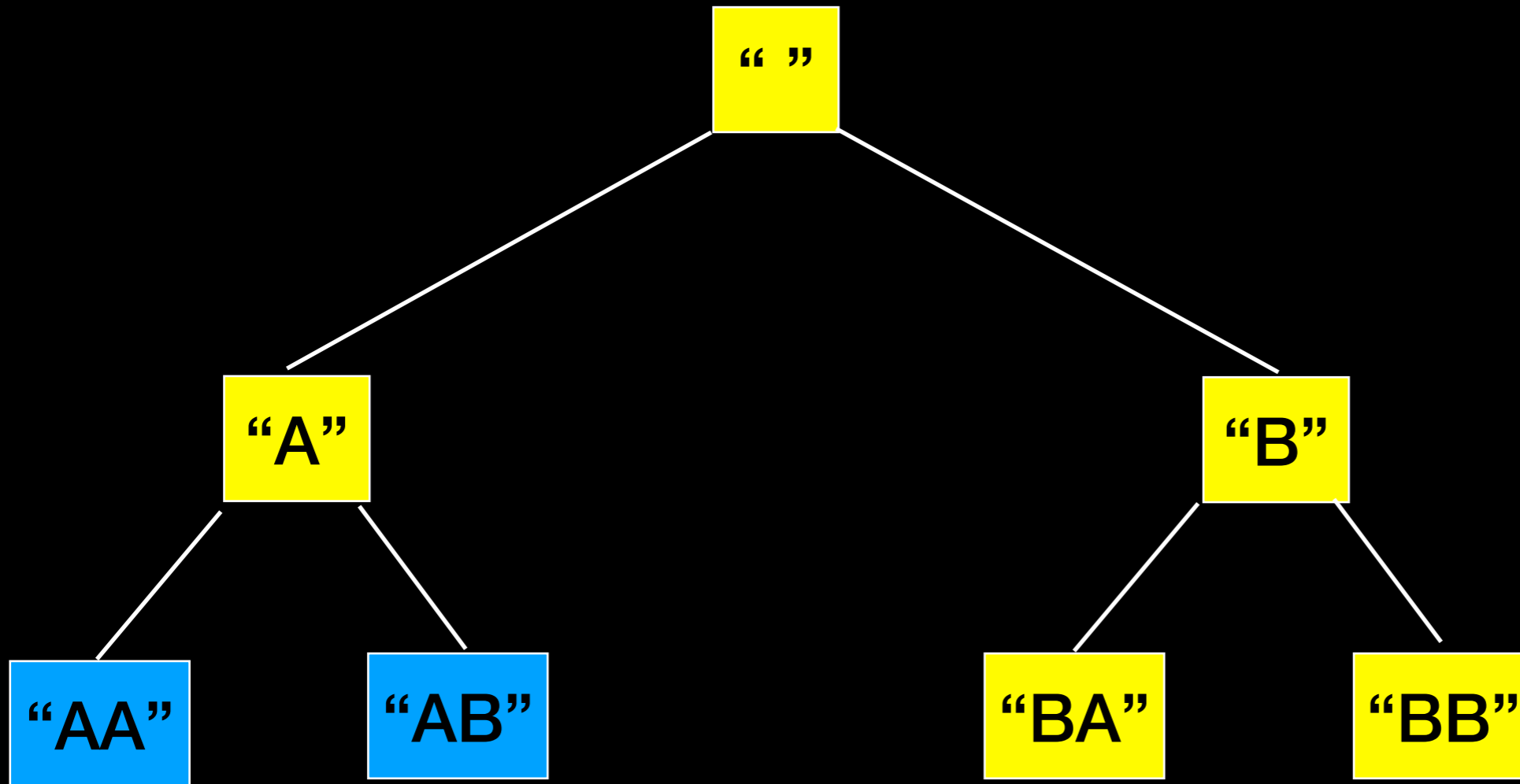
{ "", "B", "BB", "BA", "A" }

"A"

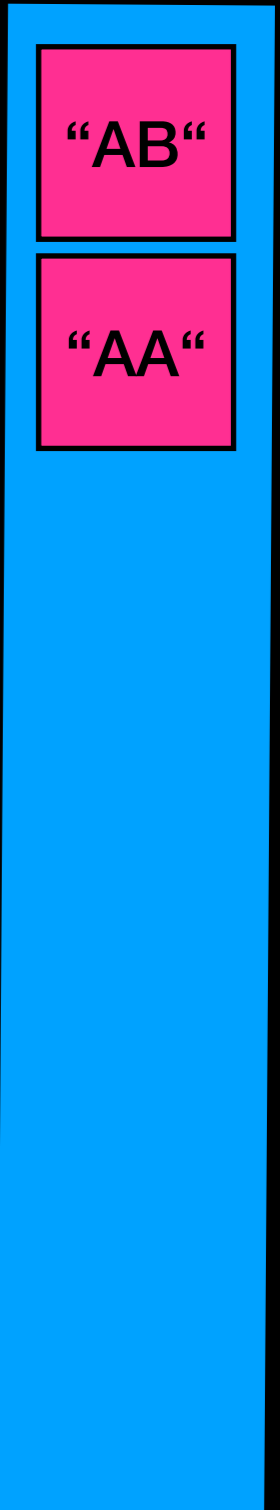
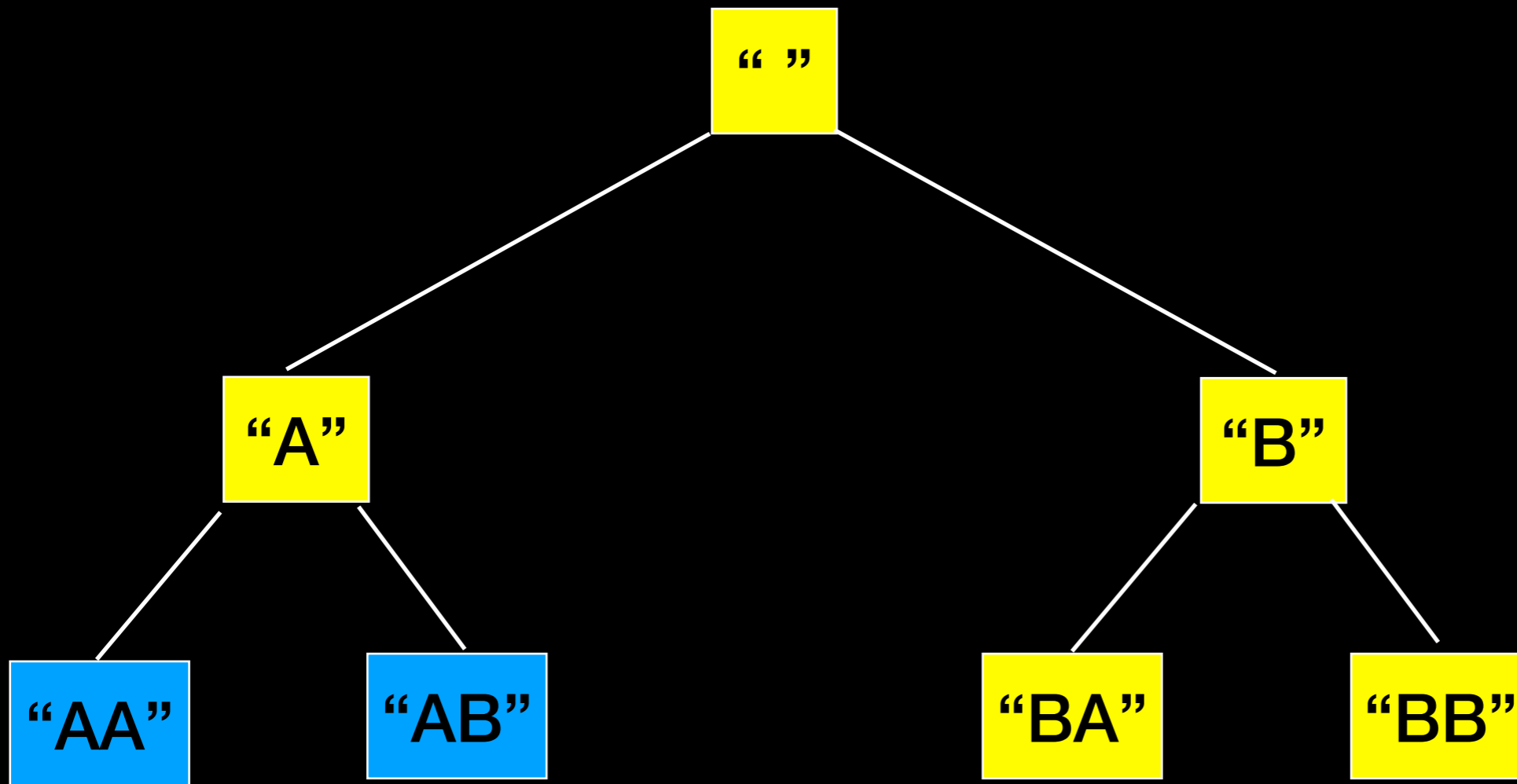


{ "", "B", "BB", "BA", "A" }

"A" "AA" "AB"

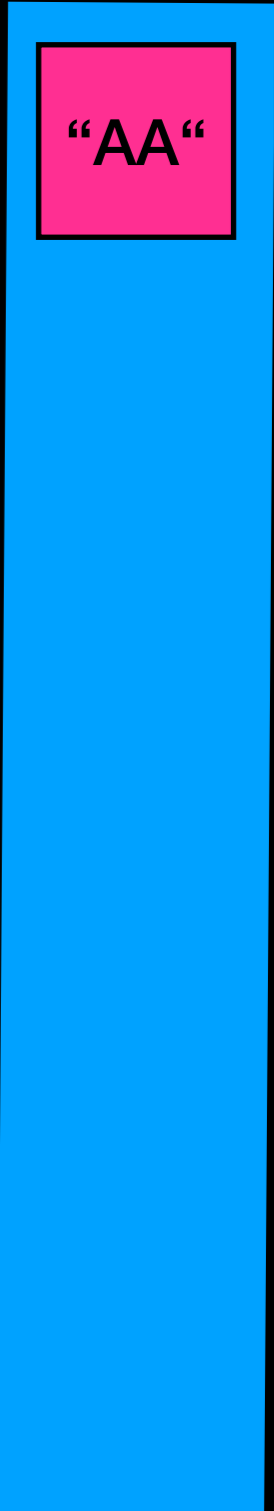
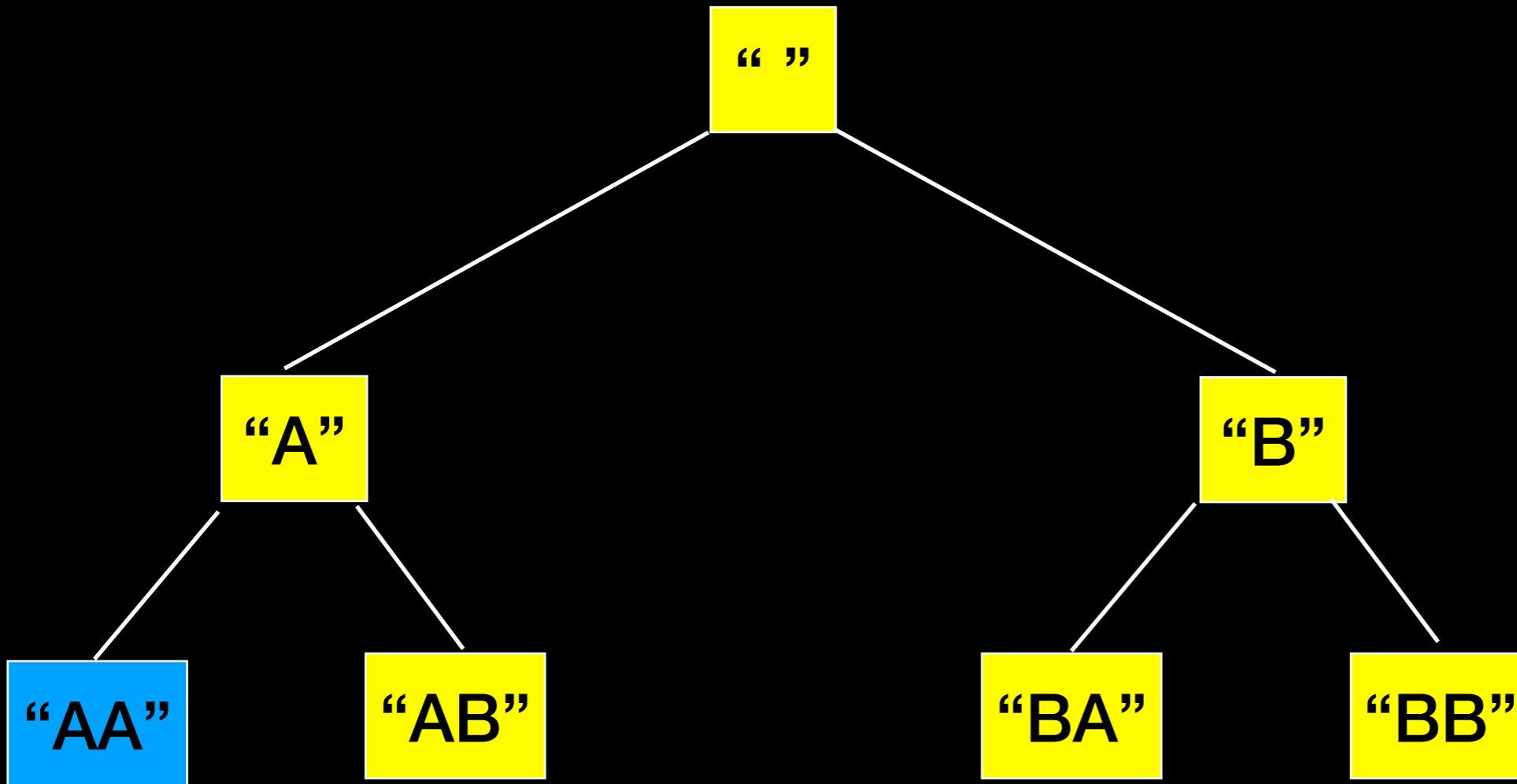


{ "", "B", "BB", "BA", "A" }



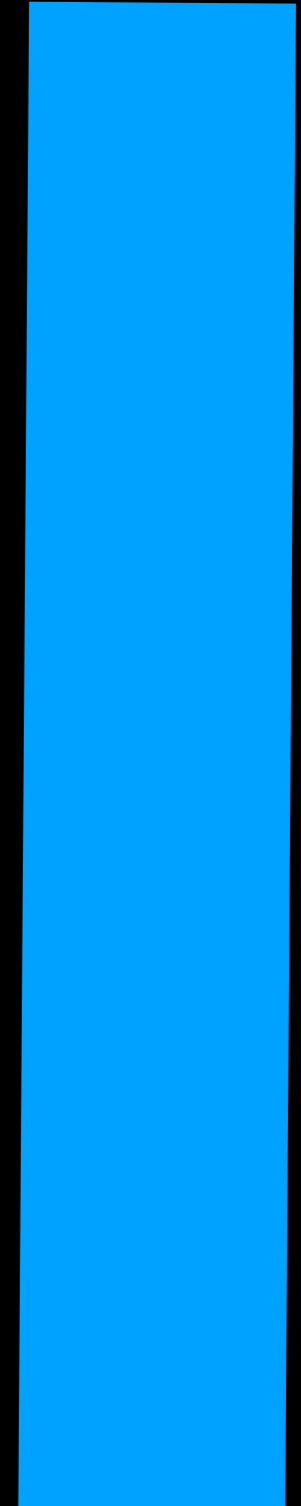
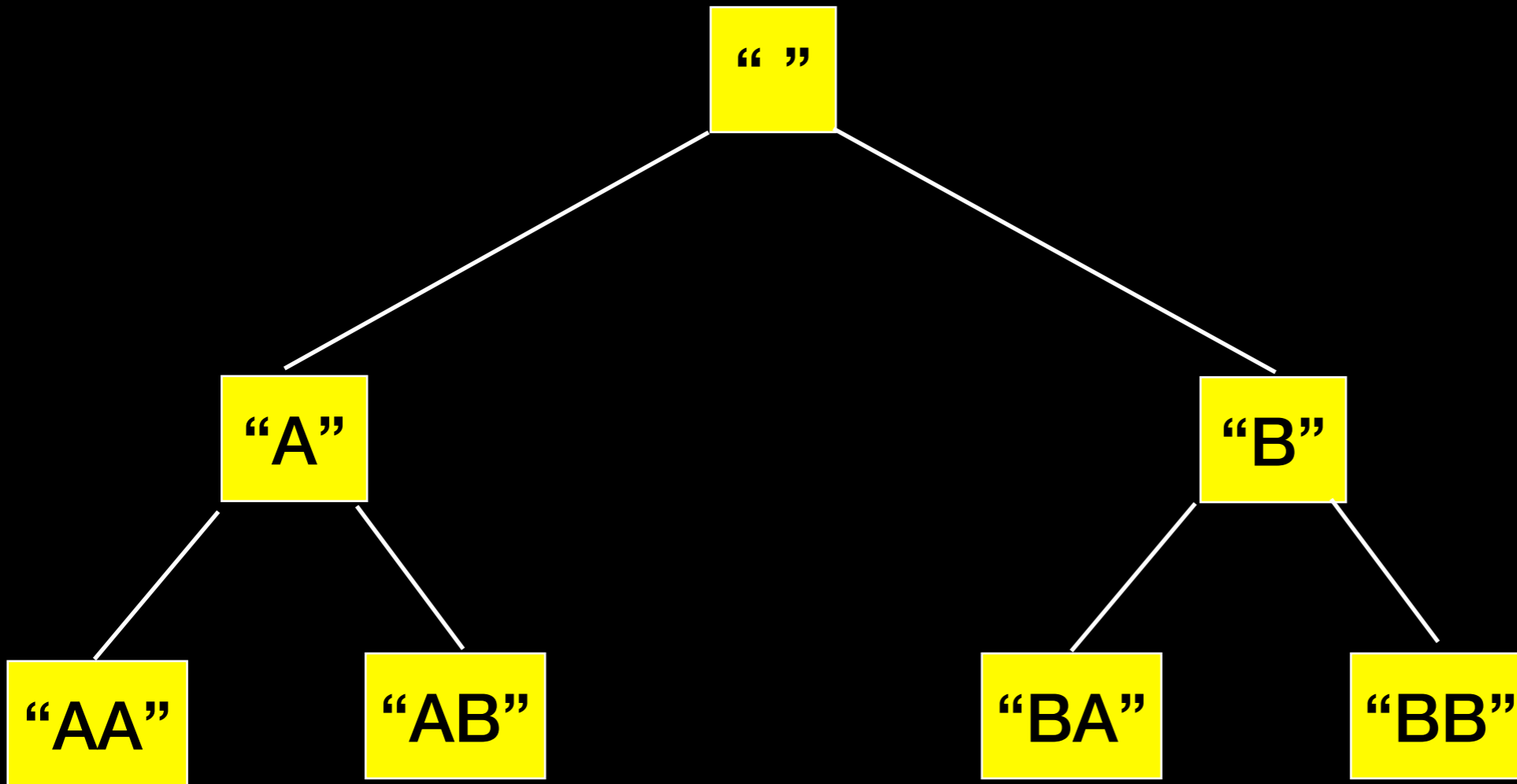
{ "", "B", "BB", "BA", "A", "AB" }

"AB"

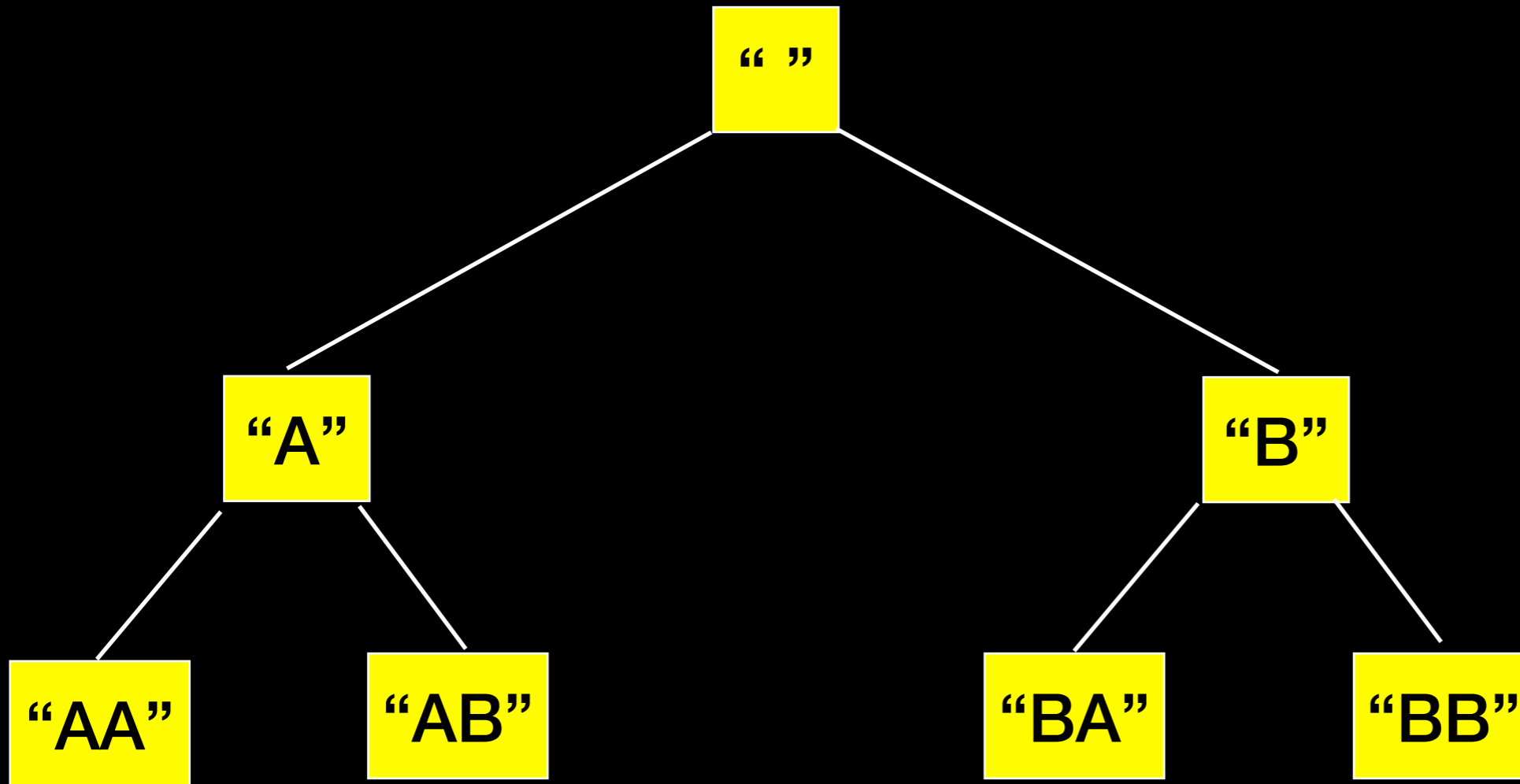


{ "", "B", "BB", "BA", "A", "AB", "AA" }

"AA"



{ "", "B", "BB", "BA", "A", "AB", "AA" }



What's the difference?

Depth-First Search

Applications

Detecting cycles in graphs

Path finding

Finding strongly connected components in graph

...

Same worst-case runtime analysis

More space efficient than previous approach

Does not explore options in increasing order of size

Comparison

Breadth-First Search
(using a queue)

Time $O(26^n)$

Space $O(26^n)$

Good for exploring options in increasing order of size when expecting to find "shallow" or "short" solution

Memory inefficient when must keep each "level" in memory

Depth-First Search
(using a stack)

Time $O(26^n)$

Space $O(n)$

Explores each option individually to max size - does NOT list options by increasing size

More memory efficient

Queue ADT

```
#ifndef QUEUE_H_
#define QUEUE_H_

template<class T>
class Queue
{

public:
    Queue();
    void enqueue(const T& new_entry); // adds an element to back queue
    void dequeue(); // removes element from front of queue
    T front() const; // returns a copy of element at the front of queue
    int size() const; // returns the number of elements in the queue
    bool isEmpty() const; // returns true if no elements in queue, false otherwise

private:
    //implementation details here

}; //end Queue

#include "Queue.cpp"
#endif // QUEUE_H_`
```

Other ADTs

Deque

Double ended queue (deque)

Can add and remove to/from front and back



Deque

Double ended queue (deque)

Can add and remove to/from front and back



Deque

Double ended queue (deque)

Can add and remove to/from front and back



Deque

Double ended queue (deque)

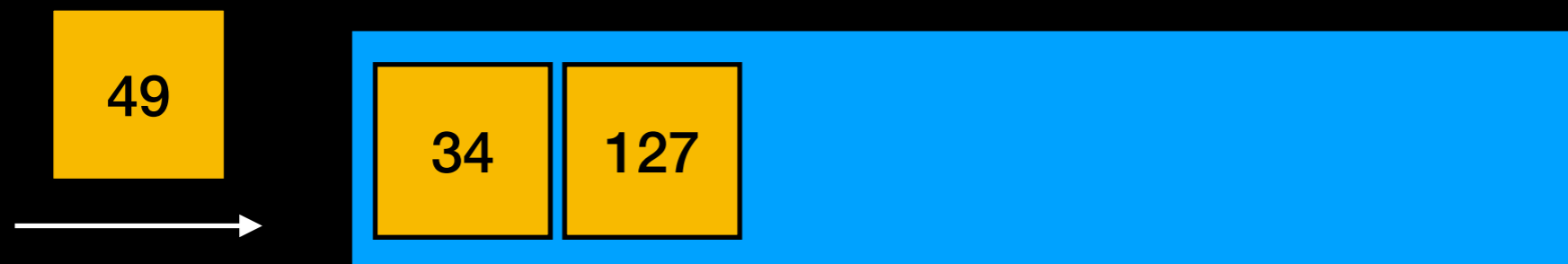
Can add and remove to/from front and back



Deque

Double ended queue (deque)

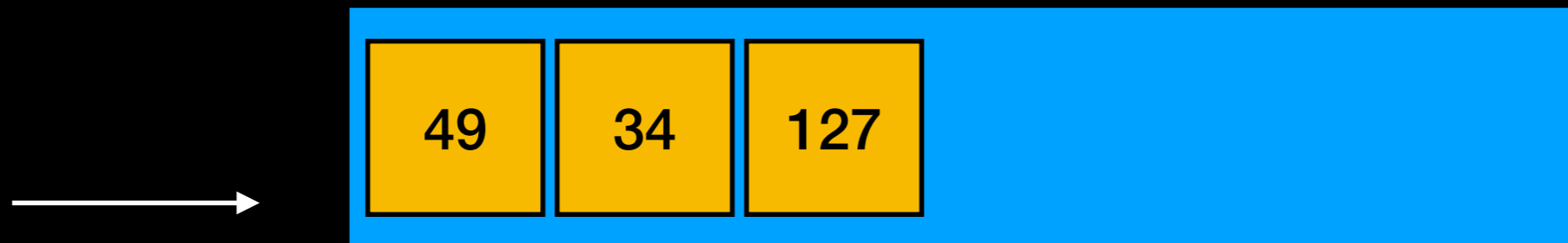
Can add and remove to/from front and back



Deque

Double ended queue (deque)

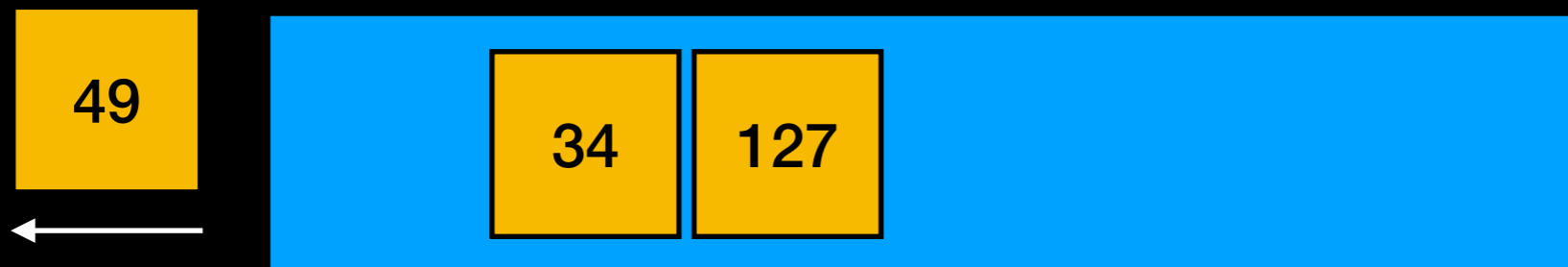
Can add and remove to/from front and back



Deque

Double ended queue (deque)

Can add and remove to/from front and back



Deque

Double ended queue (deque)

Can add and remove to/from front and back



Deque

Double ended queue (deque)

Can add and remove to/from front and back



Deque

In STL :

- does not use contiguous memory
- more complex to implement (keep track of memory blocks)
- grows more efficiently than vector

Deque

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In STL stack and queue are *adapters* of deque

Deque

In STL :

- does not use contiguous memory
- more complex to implement (keep track of memory blocks)
- grows more efficiently than vector

In STL stack and queue are *adapters* of deque

STL standardized the use of language "push" and "pop", adapting with "push_back", "push_front" etc. for all containers

Priority Queue

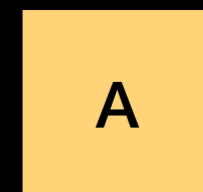
Low Priority



High Priority



A queue of items "sorted" by priority



Priority Queue

Low Priority



High Priority



A queue of items "sorted" by priority



Priority Queue

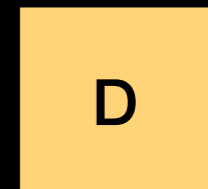
Low Priority



High Priority



A queue of items "sorted" by priority



Priority Queue

Low Priority



High Priority



A queue of items "sorted" by priority



Priority Queue

Low Priority



High Priority



A queue of items "sorted" by priority



Priority Queue

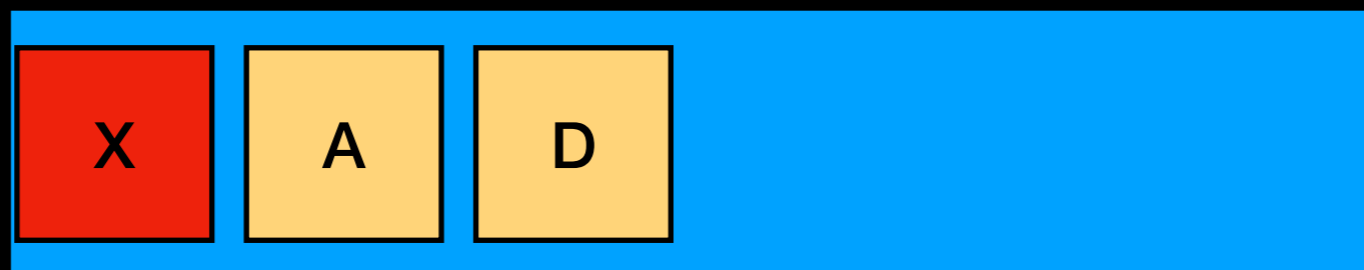
Low Priority



High Priority



A queue of items "sorted" by priority



Priority Queue

Low Priority



High Priority



A queue of items "sorted" by priority



Priority Queue

Low Priority



High Priority



A queue of items "sorted" by priority

If value indicates priority, it amounts to a sorted list that accesses/removes the "highest" items first



Priority Queue

Orders elements by priority => removing an element will return the element with highest priority value

Elements with same priority kept in queue order (in some implementations)

Priority Queue

Spoiler Alert!!!!

Often implemented with a **Heap**

Will tell you what it is in a few weeks... but here is another example of ADT vs data structure

Explore the STL

It's time to get to know it!!!

C++ Interlude 8 in your textbook

https://en.cppreference.com/w/cpp/standard_library

<https://en.cppreference.com/w/cpp/container>

<https://en.cppreference.com/w/cpp/algorithm>

You should use STL stack and queue for Project 6

Explore as you learn about new ADTs and algorithms.

Main Components

Containers

Algorithms

Functions

Iterators

Main Components

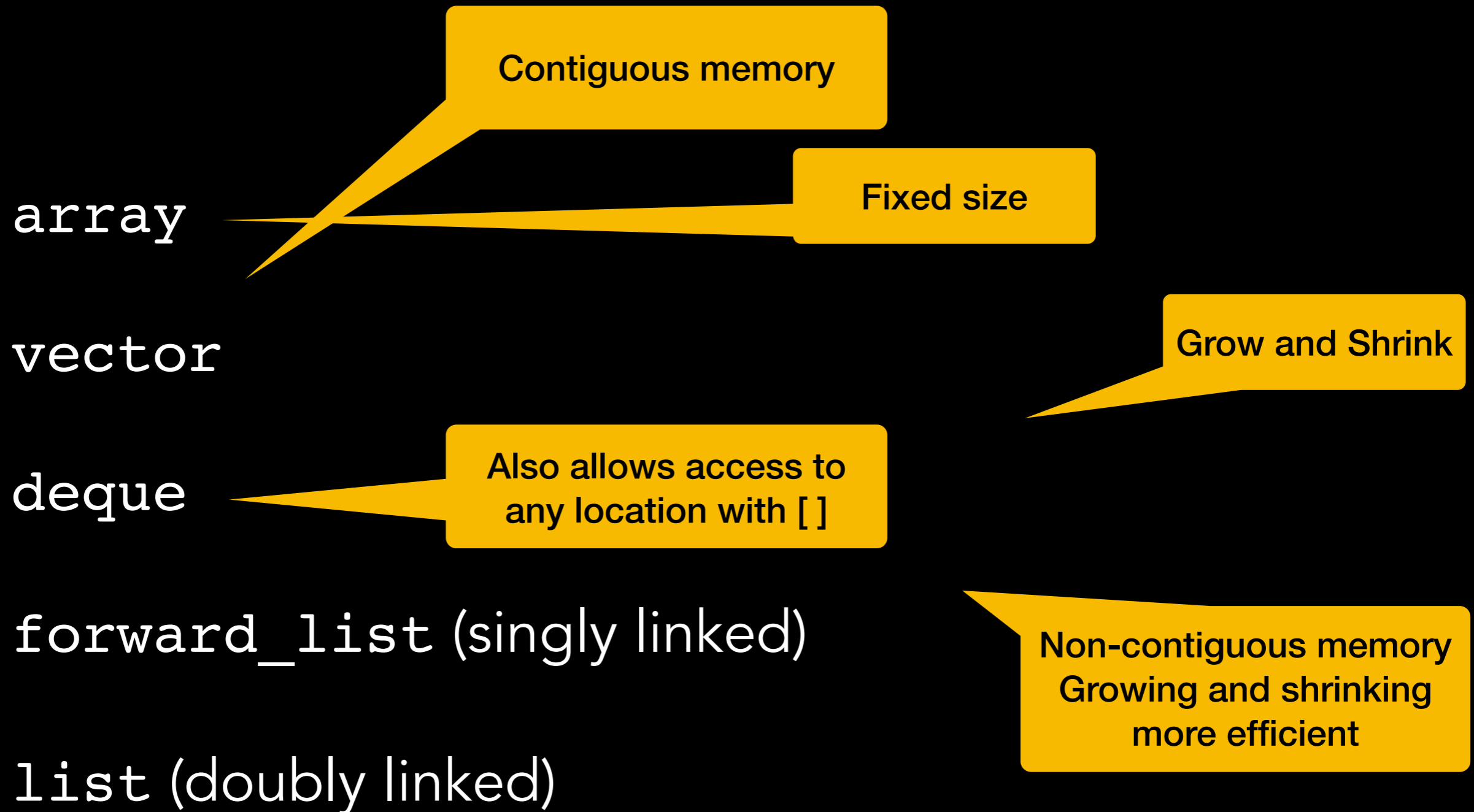
Containers

Algorithms

Functions

Iterators

Sequence Containers



Container Adaptors

Impose a different interface for the underlying container

`stack`

`queue`

`priority_queue`

For Project 6

```
#include <stack>
#include <queue>

std::queue<Attack> attack_queue_;
std::stack<Creature*> alien_stack_;
std::stack<Creature*> undead_stack_;
std::stack<Creature*> mystical_stack_;
std::stack<Creature*> unknown_stack_;

attack_queue_.push(attack);
//uses stack language but always adds
// to the back of the queue

attack_queue_.pop();
//uses stack language but always removes from the
//front of the queue, does NOT return
// the popped item
```

Algorithms

```
#include <algorithm>
```

Search and Compare Algorithms examples

```
for_each() // applies a function to a range in container  
count() // counts the occurrences of an item within a range  
max_element() // returns the max value within a range
```

Sequence Modification Algorithms examples

```
copy() //copies items within a range starting at given position  
       within same or different container  
fill() // sets all entries within a range to give value
```

Sorting and Searching Algorithms examples

```
sort() // sorts entries within a range in ascending order –  
       typically some variation of QuickSort  
stable_sort() // "" – typically MergeSort may vary  
binary_search() // determines if an item exist in a given range  
                in a sorted container
```

... much more!!!