

# Heaps



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# Announcements

Please fill out the teacher evaluation form,  
thank you!

- Smartphone: [www.hunter.cuny.edu/mobilete](http://www.hunter.cuny.edu/mobilete)
- Computer: [www.hunter.cuny.edu/te](http://www.hunter.cuny.edu/te)

# Announcements

Focus group rescheduled, please see Blackboard announcement to register

Fill out paper survey to get credit for the workshops!

# Announcements

- Final Exam MONDAY May 20, 9-11am
- Review Tuesday May 14
  - **Student Led**
  - Come with questions

# Heap

A **Heap** is a complete binary tree that is either

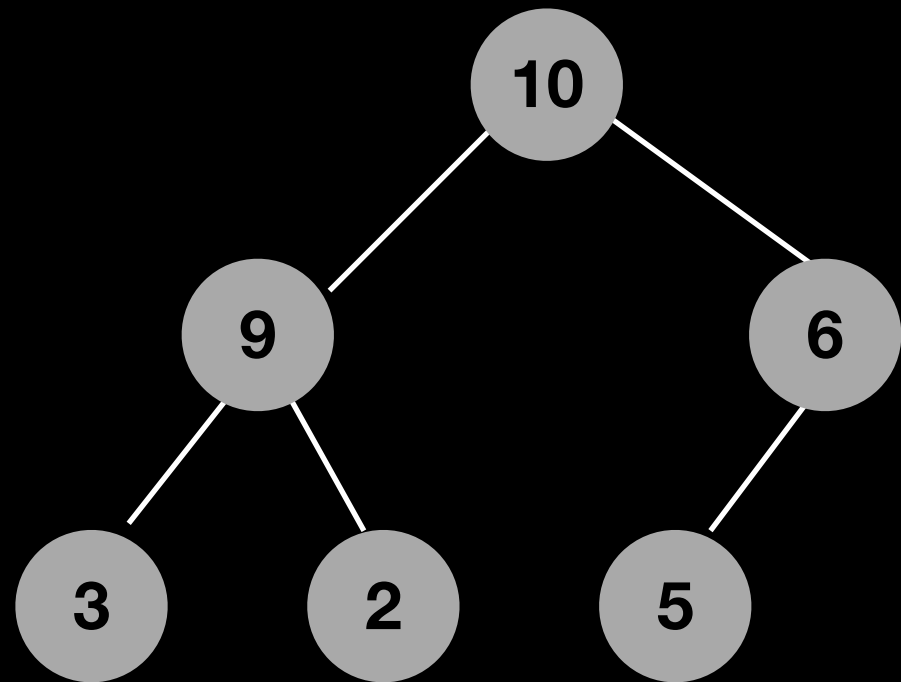
- Empty or
- Its **root** contains a value  $\geq$  (or  $\leq$ ) both of its **children** and has **heaps as subtrees**

# Heap

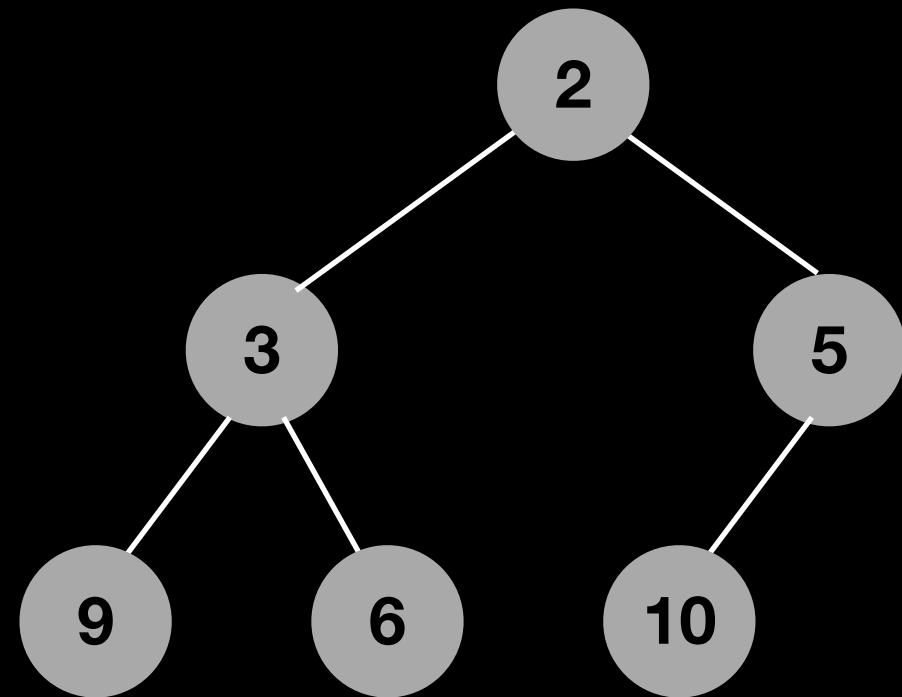
A special binary tree:

- **Ordered** in a weaker sense
- Always a **complete** binary tree

MaxHeap



MinHeap



# Implementation

How would you implement it???

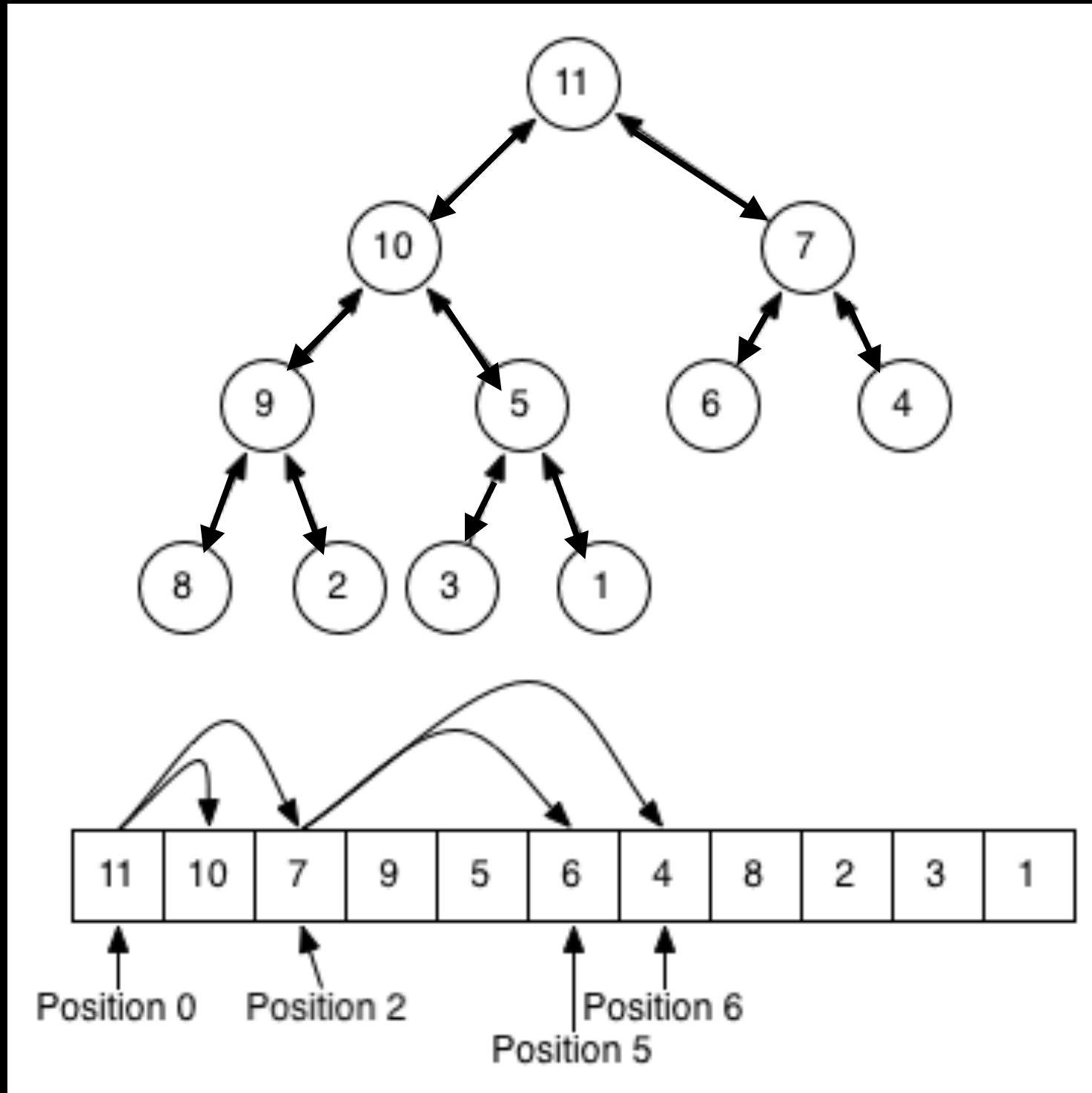


# Implementation

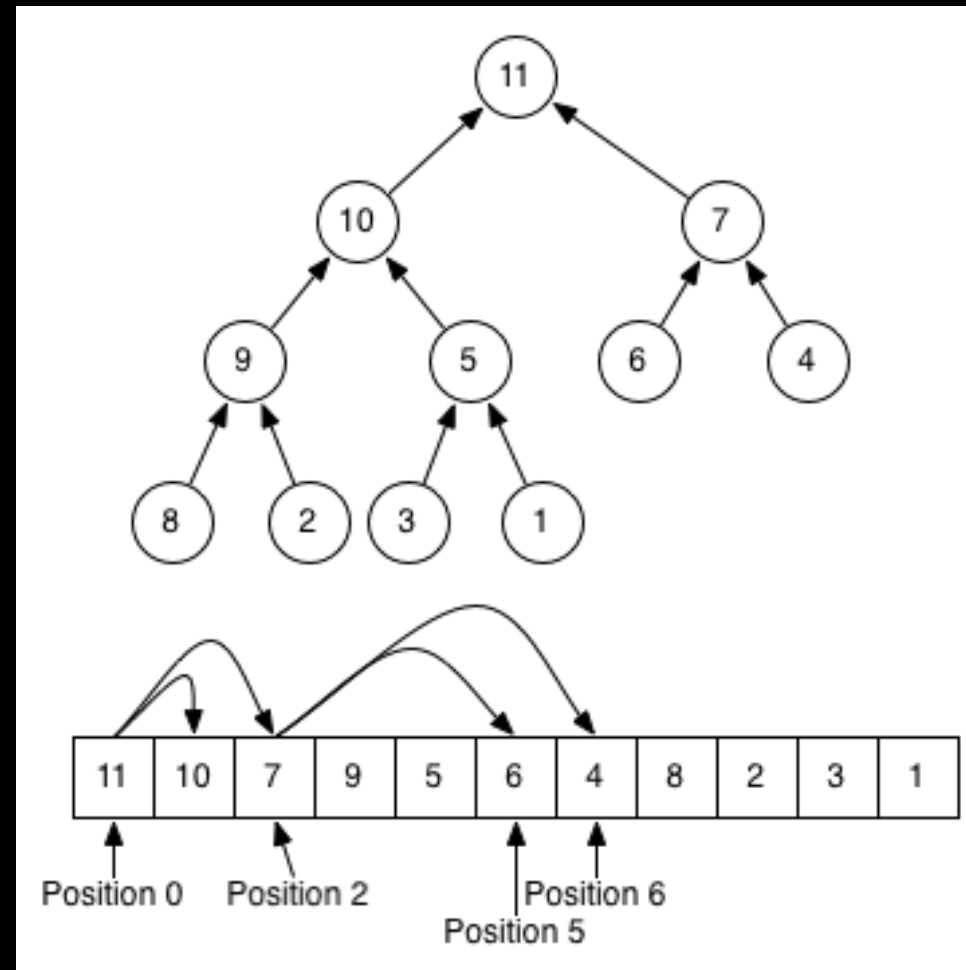
How would you implement it???

Insight: it is **always complete**

# Implementation



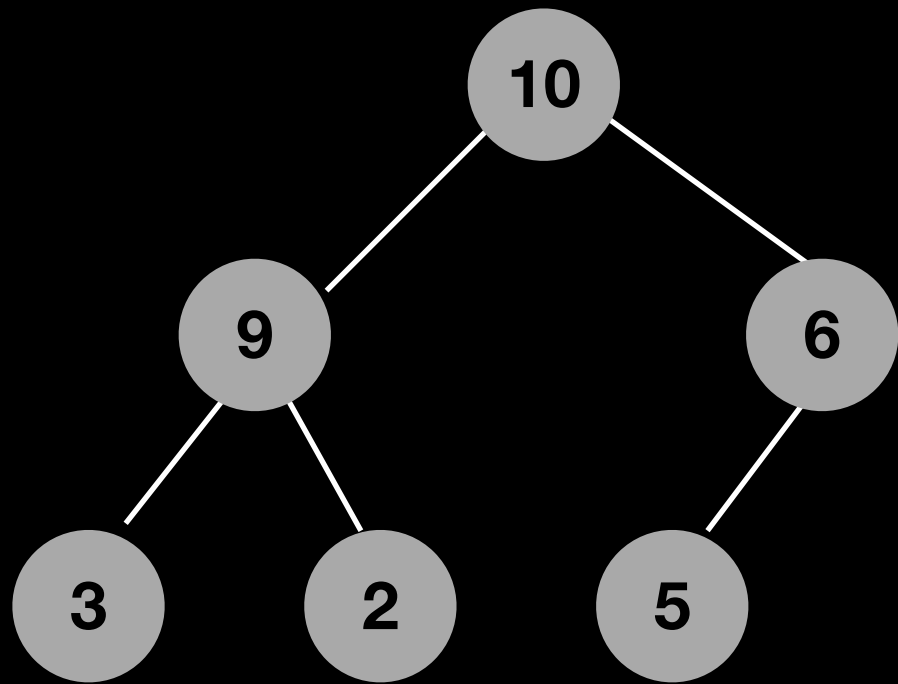
# Implementation



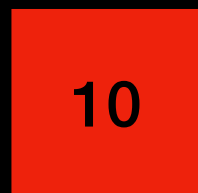
```
root_ = items_[0]
items_[i] left_child = items_[2 * i + 1]
items_[i] right_child = items_[ 2 * i + 2]
items_[i] parent = items_[(i-1)//2]
```

floor division

MaxHeap



Priority Queue



# Retrieve

Can only retrieve max/min item

Stored at root

$O(1)$

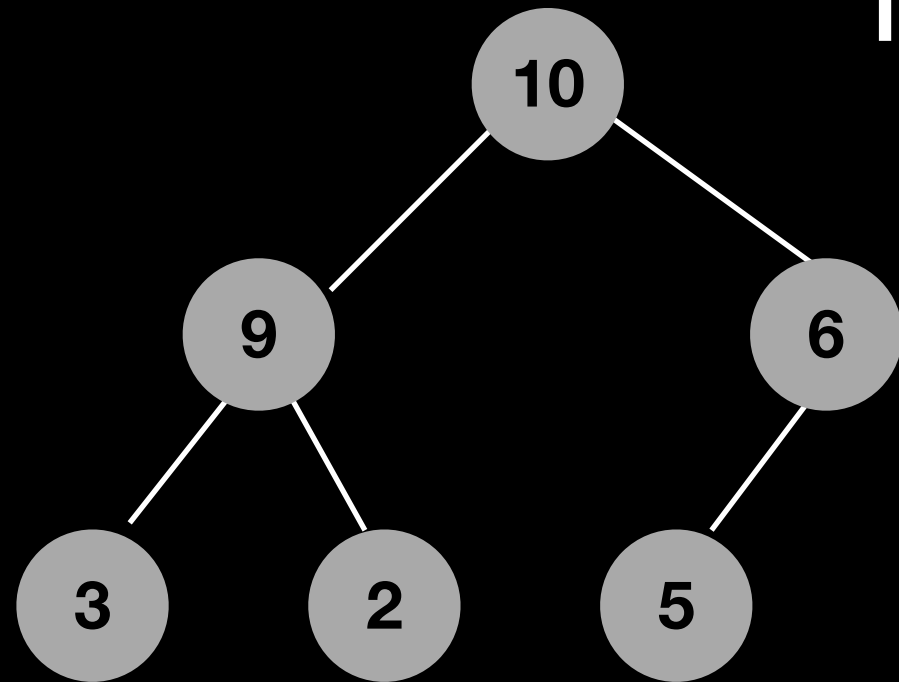
# Remove

Remove max/min item (the root)

Must retain Heap

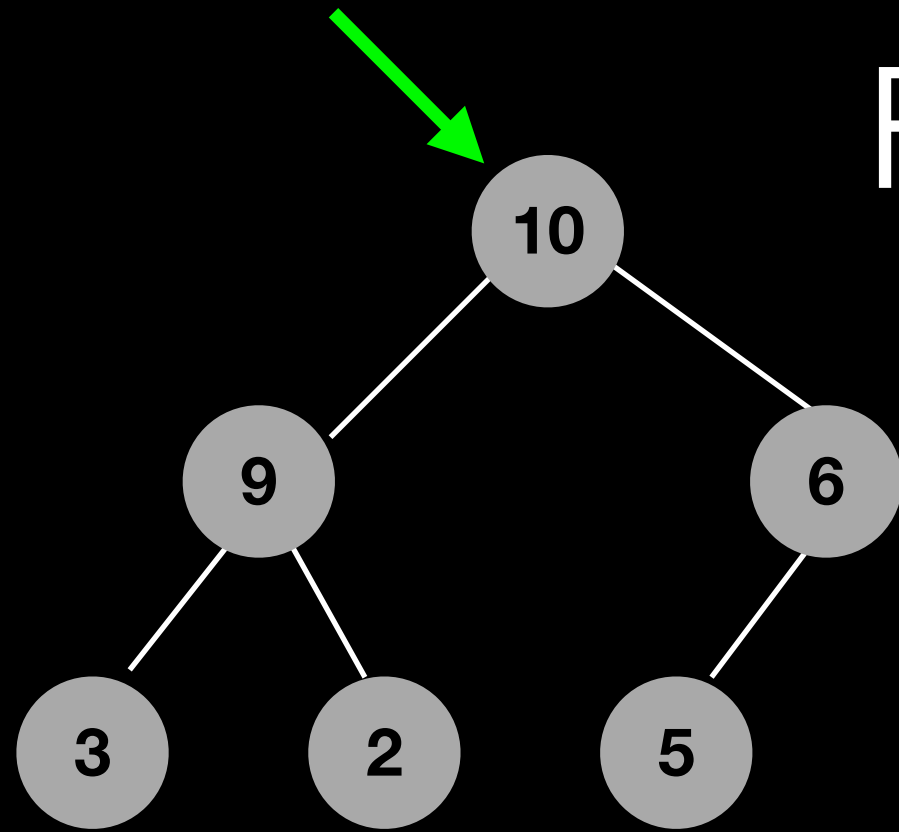
- Heap ordering property
- Complete

# Remove



**What element do we remove?**

# Remove

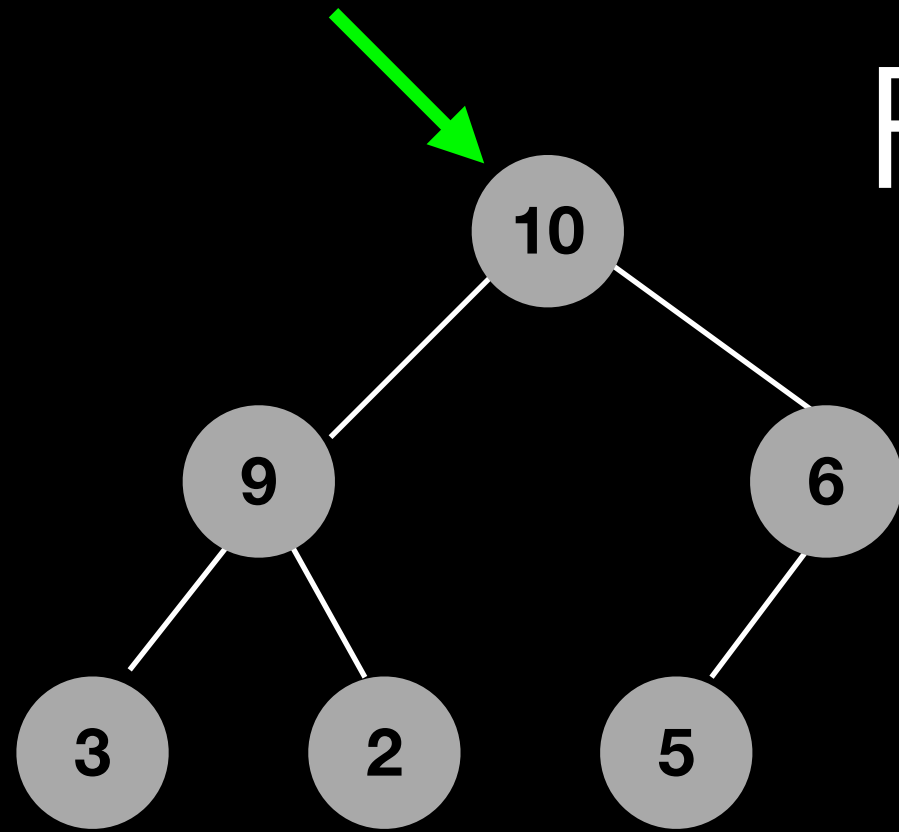


**What element do we remove?**

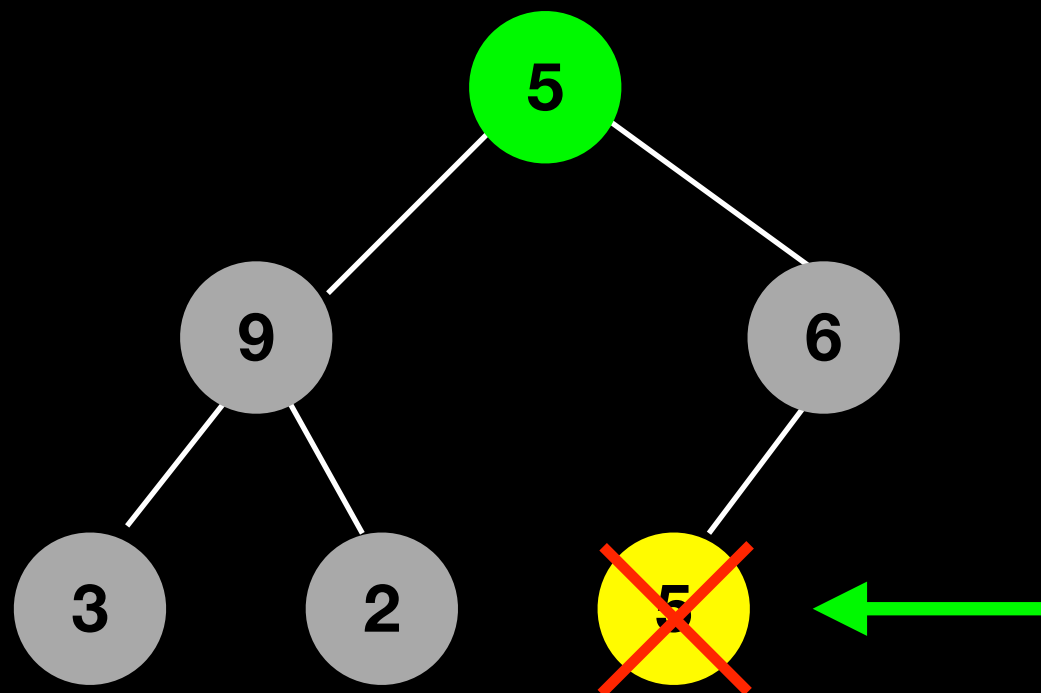
**What node do we remove?**



# Remove

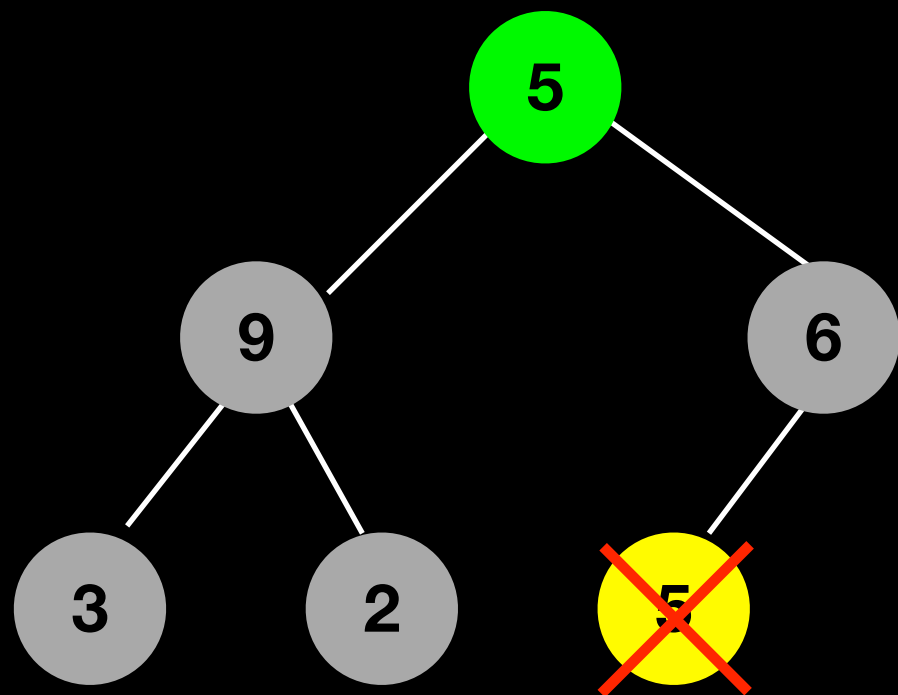
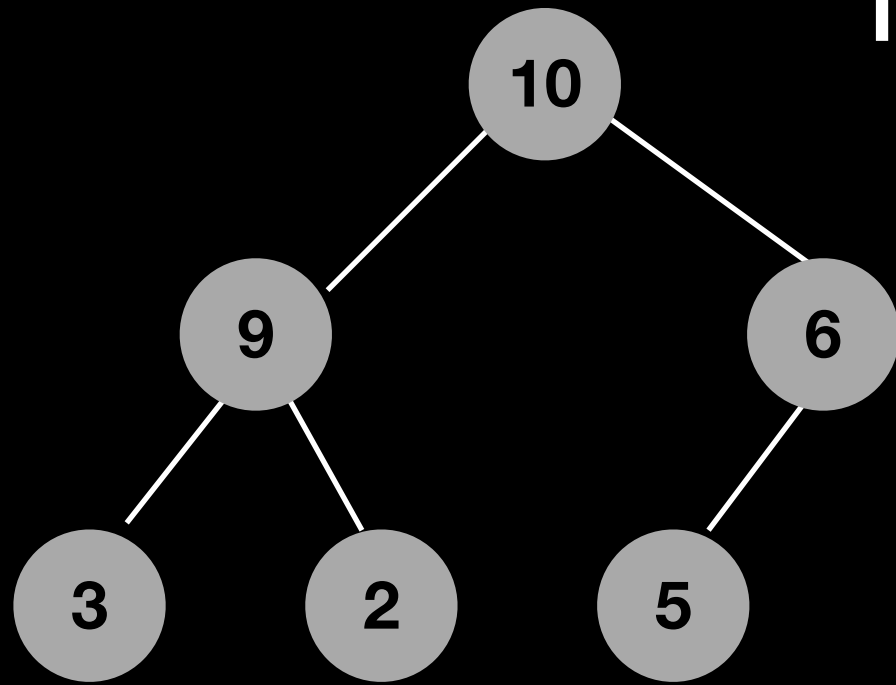


What element do we remove?

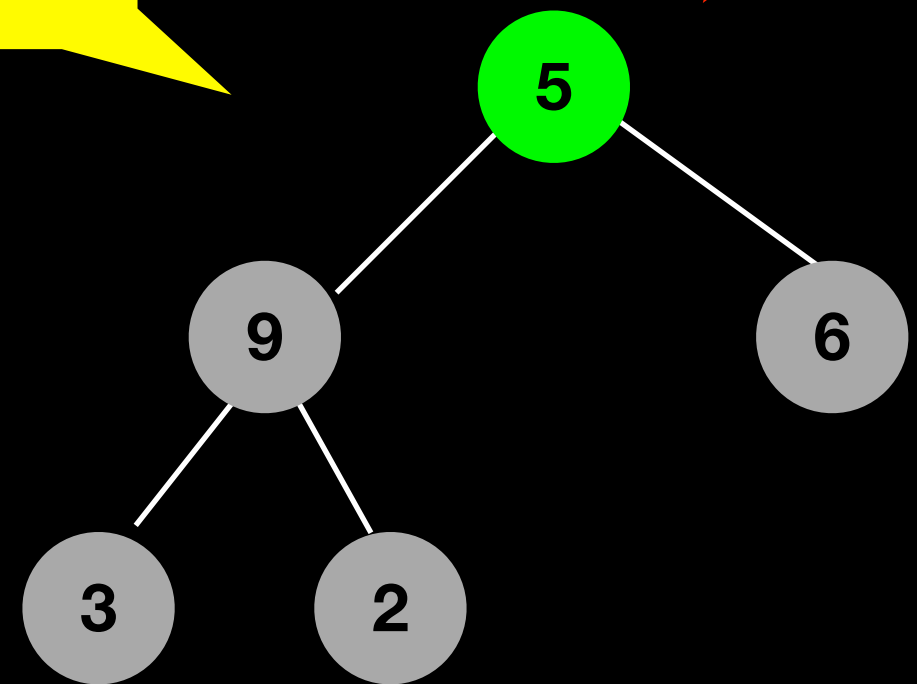


Remove this node form complete tree

# Remove



Complete

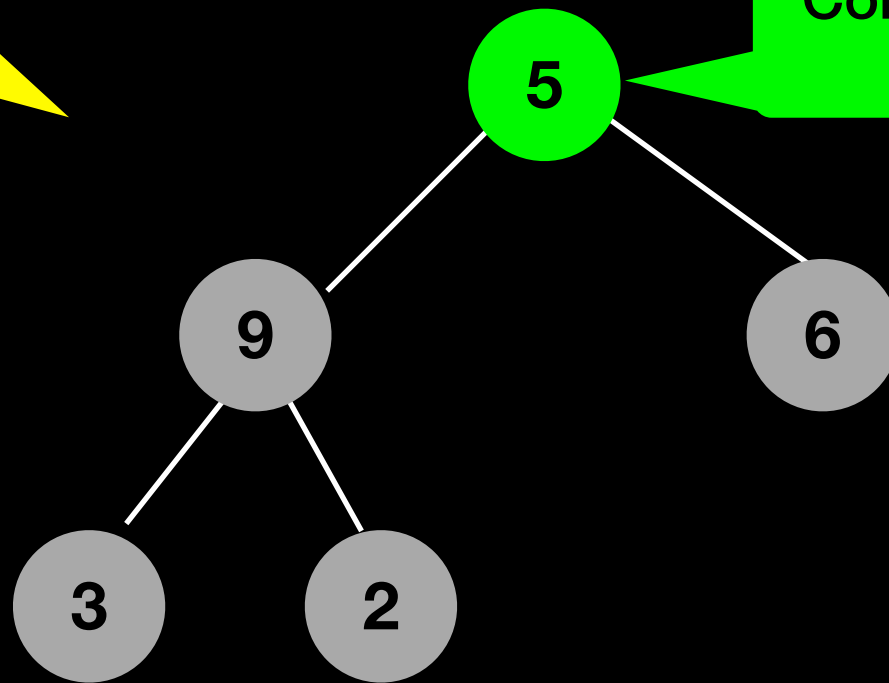


Not Heap

heapRebuild

# Remove

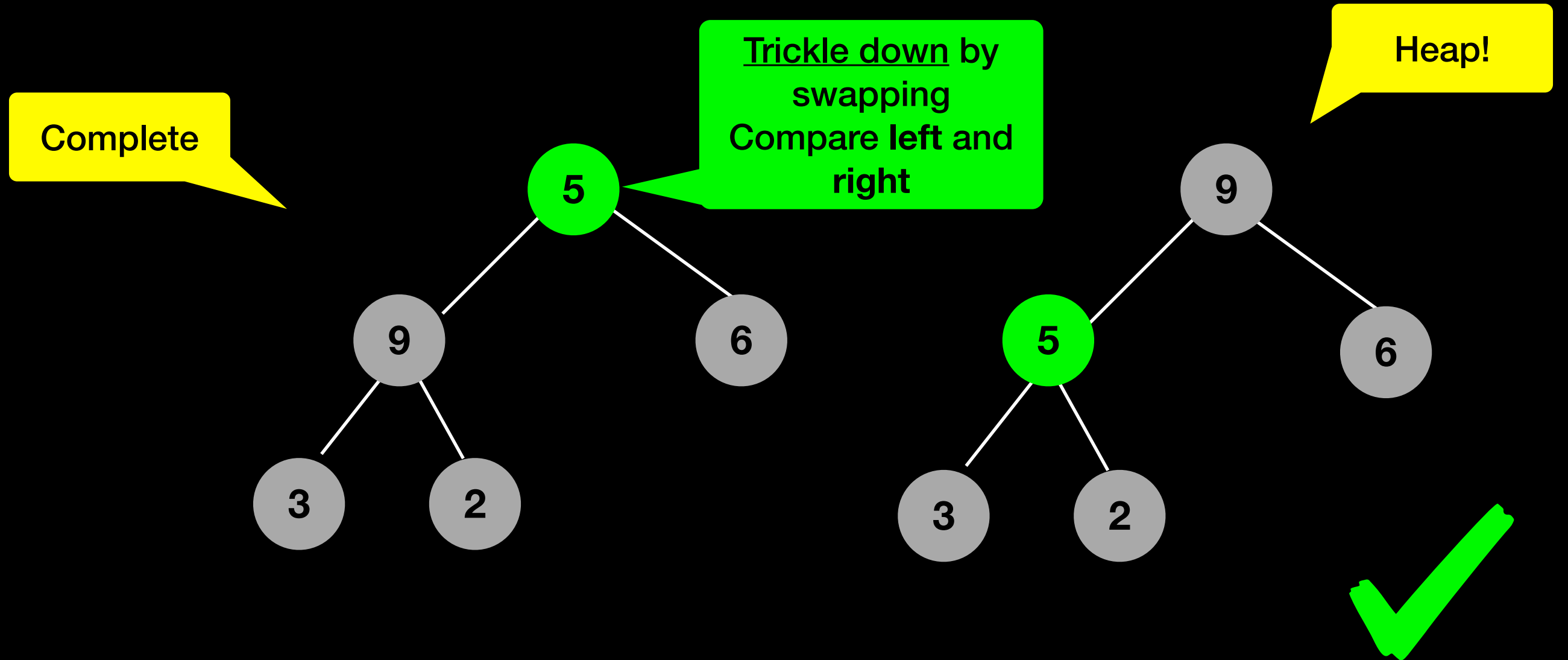
Complete



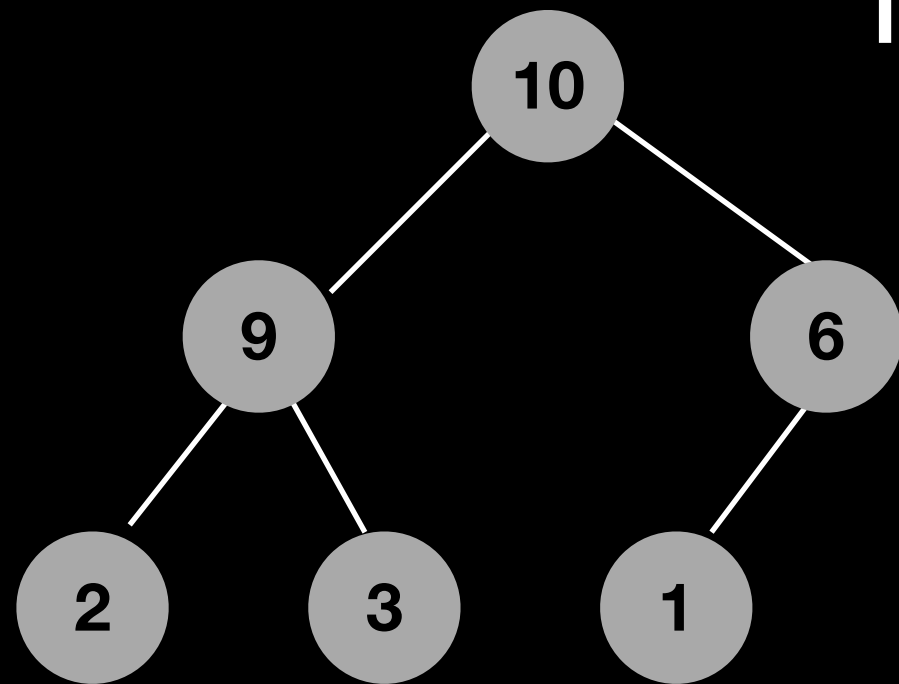
Trickle down by  
swapping  
Compare left and  
right

heapRebuild

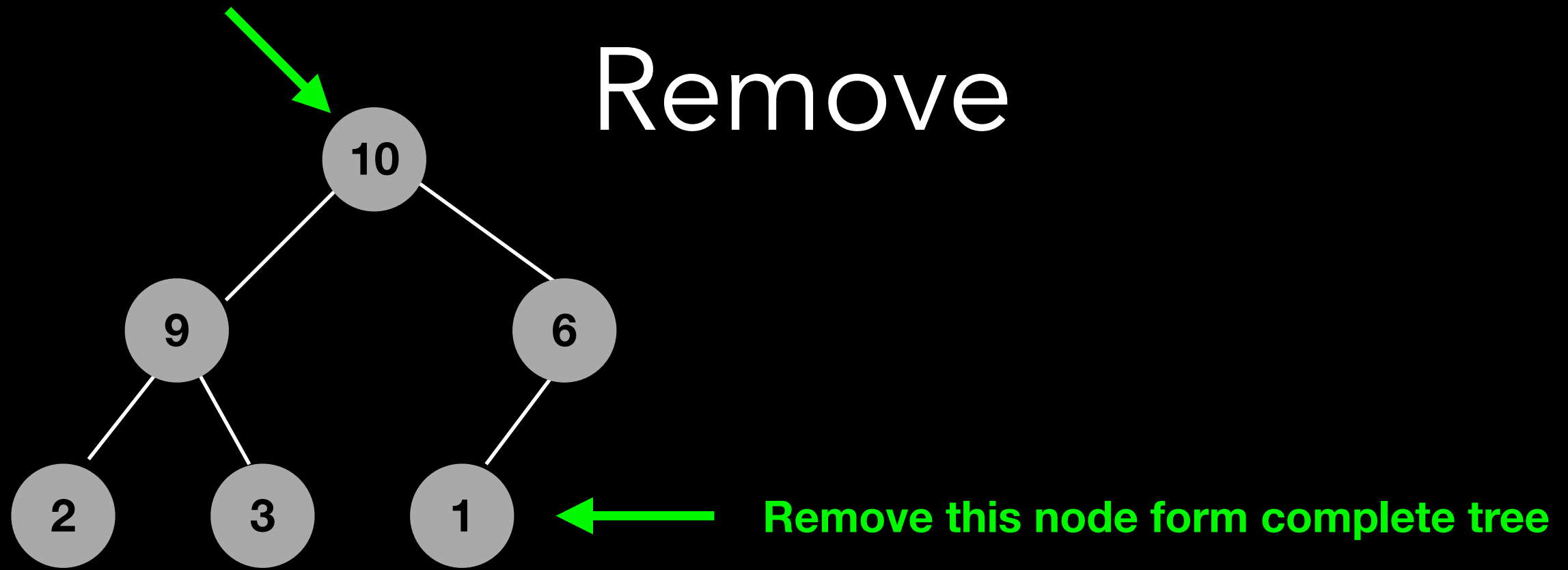
# Remove



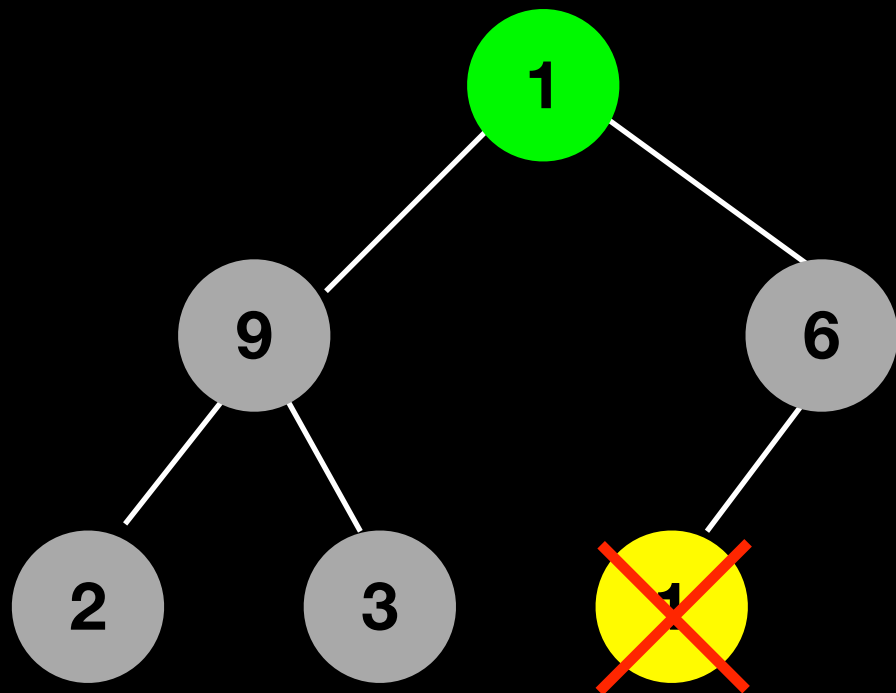
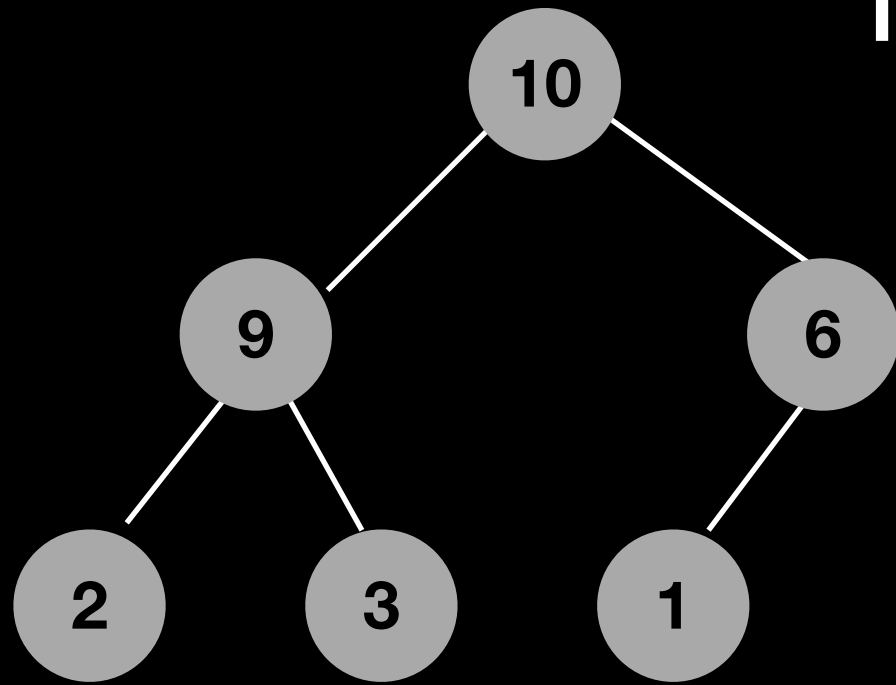
# Remove



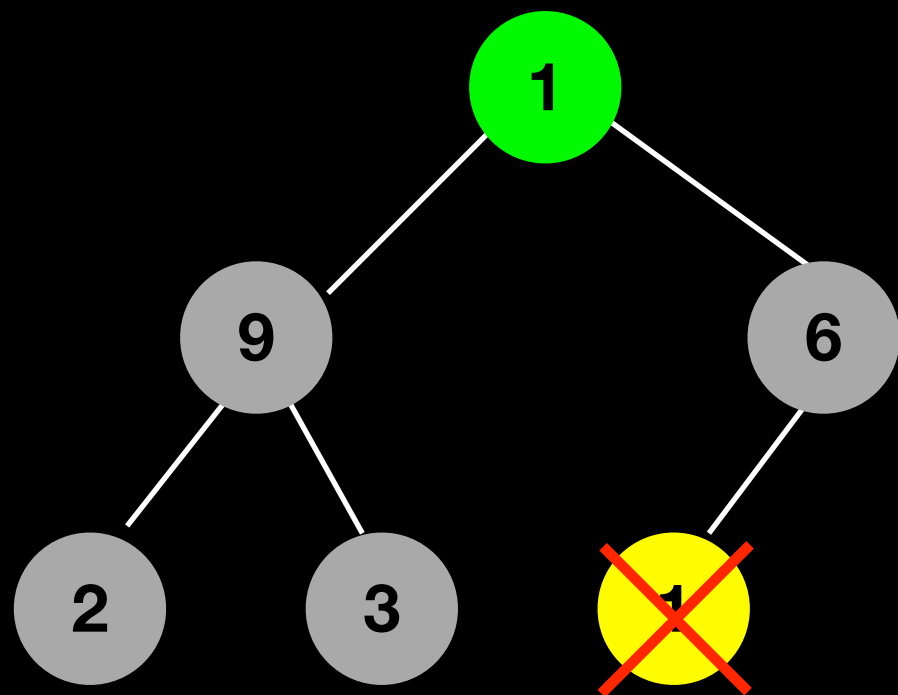
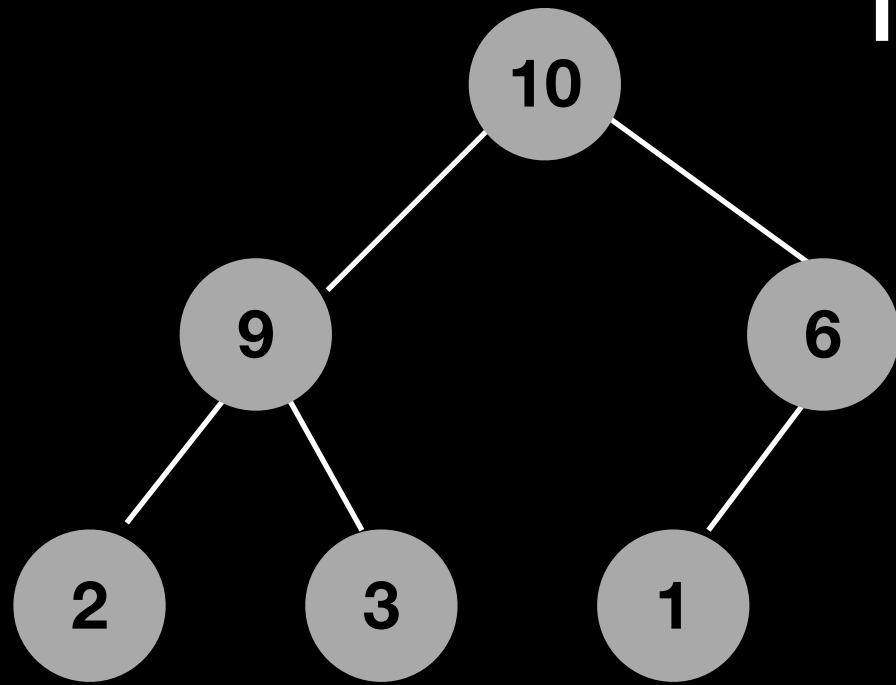
# Remove



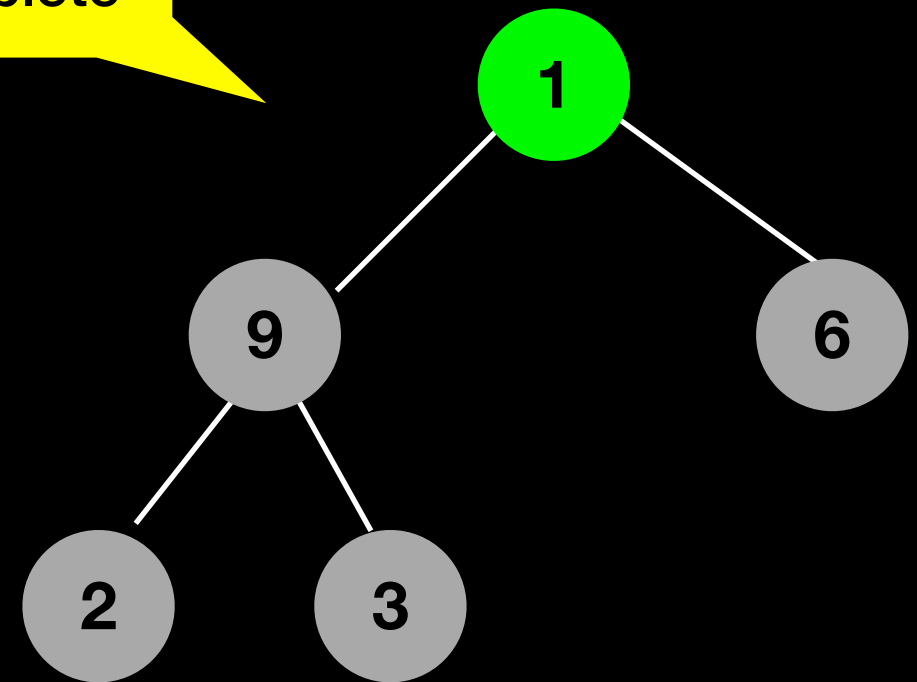
# Remove



# Remove



Complete



Not Heap

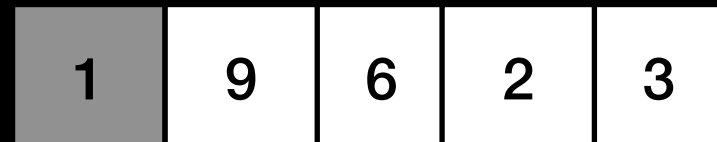
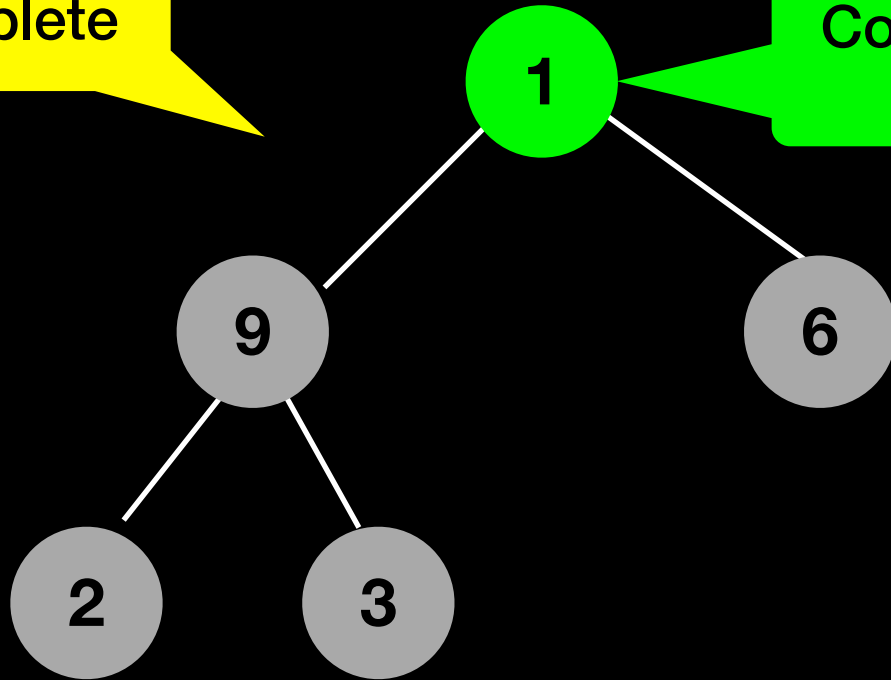


heapRebuild

# Remove

Complete

Trickle down by  
swapping  
Compare left and  
right

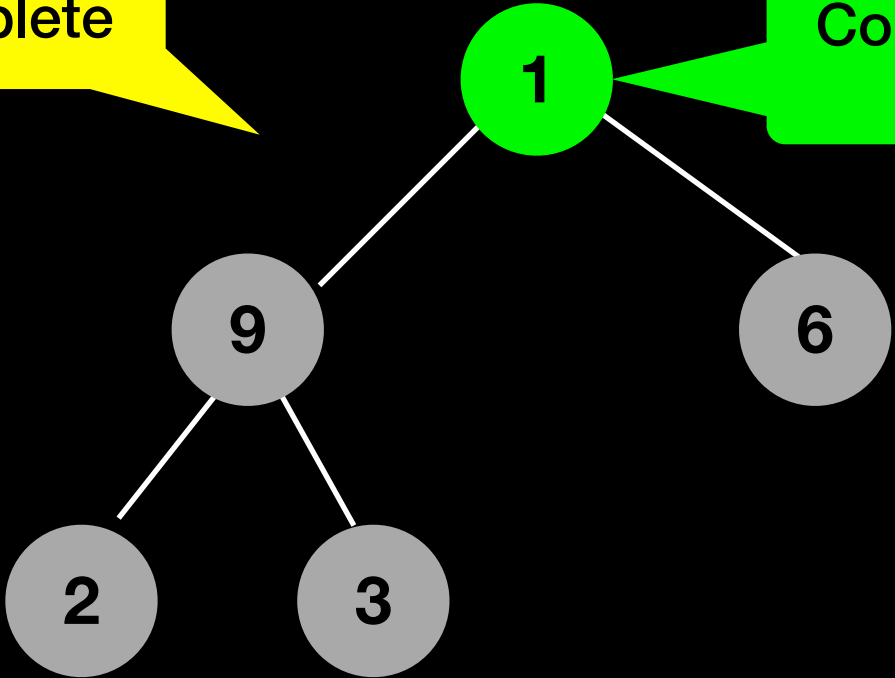


```
items_[i] left_child = items_[2 * i + 1]
items_[i] right_child = items_[2 * i + 2]
```

heapRebuild

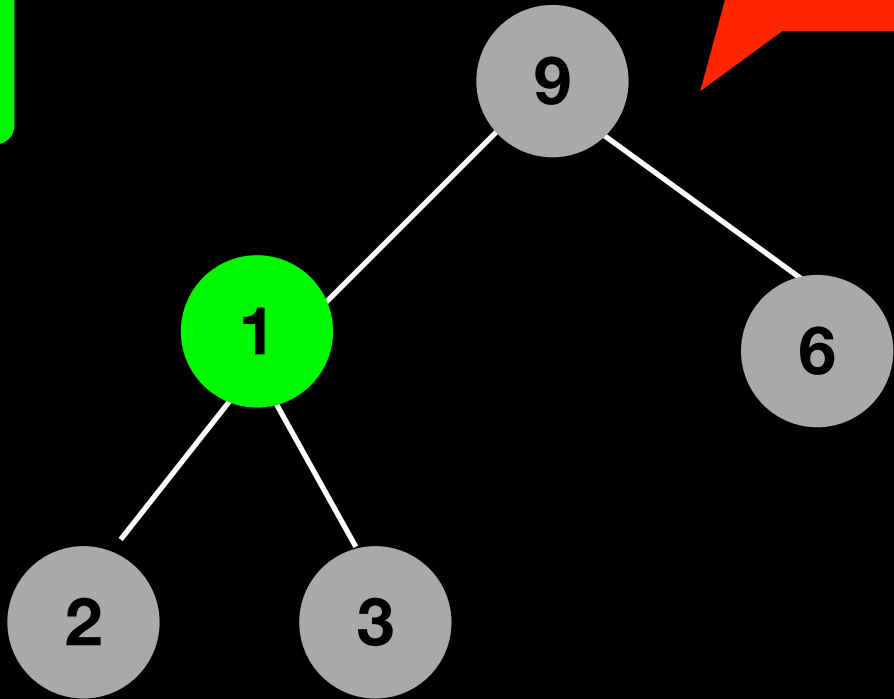
# Remove

Complete



Trickle down by swapping  
Compare left and right

Not Heap



1	9	6	2	3
1	9	6	2	3

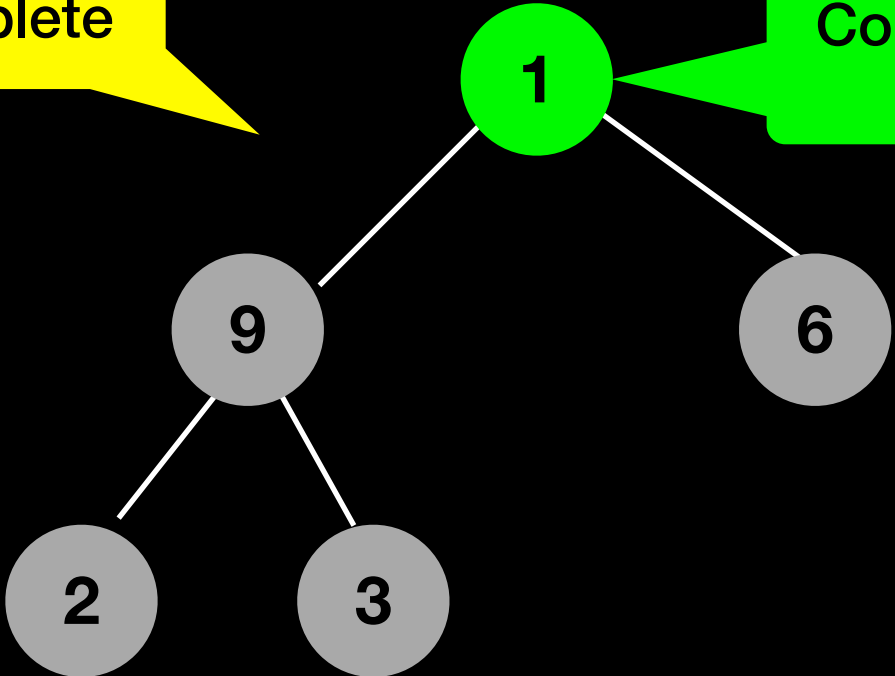
```

items_[i] left_child = items_[2 * i + 1]
items_[i] right_child = items_[2 * i + 2]
  
```

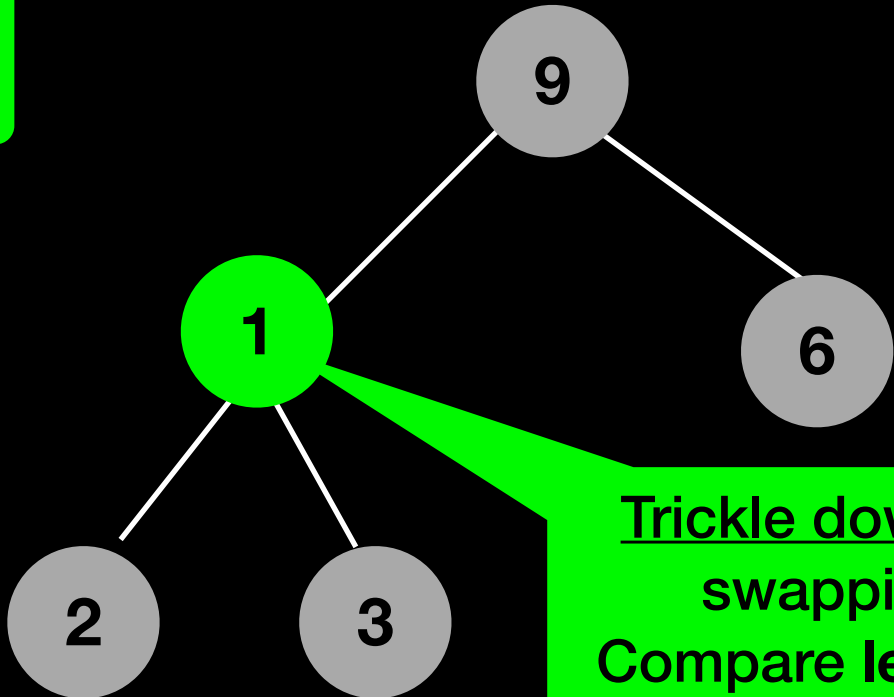
heapRebuild

# Remove

Complete



Trickle down by swapping  
Compare left and right



Trickle down by swapping  
Compare left and right

1	9	6	2	3
1	9	6	2	3

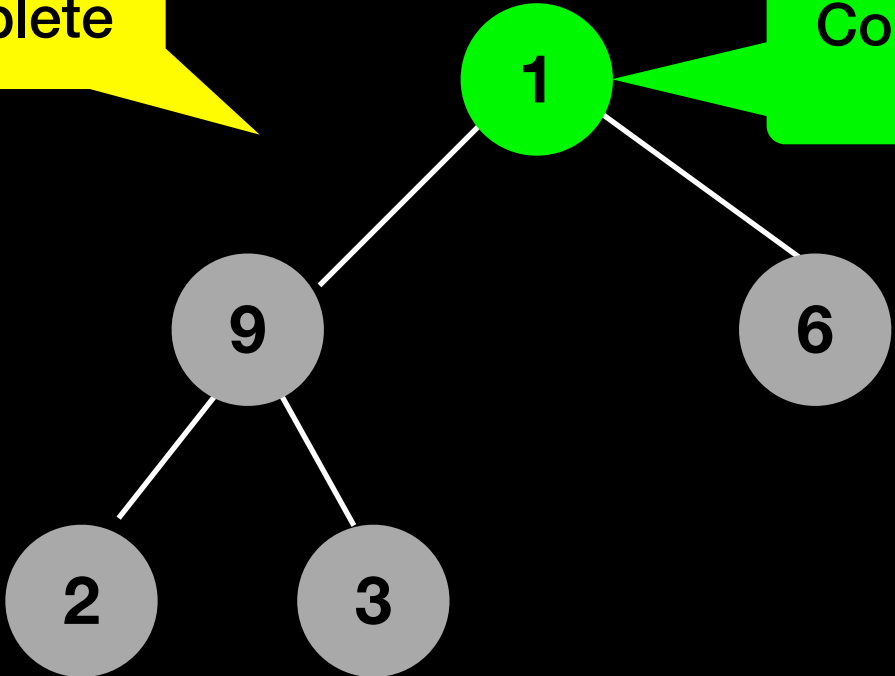
```

items_[i] left_child = items_[2 * i + 1]
items_[i] right_child = items_[2 * i + 2]
  
```

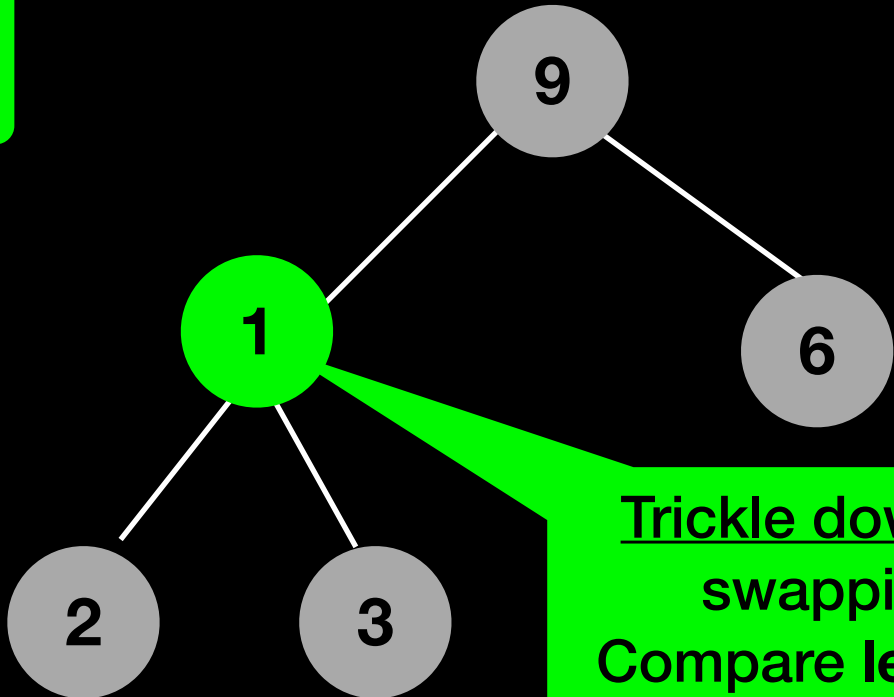
heapRebuild

# Remove

Complete



Trickle down by swapping  
Compare left and right



Trickle down by swapping  
Compare left and right

1	9	6	2	3
1	9	6	2	3
9	1	6	2	3

```

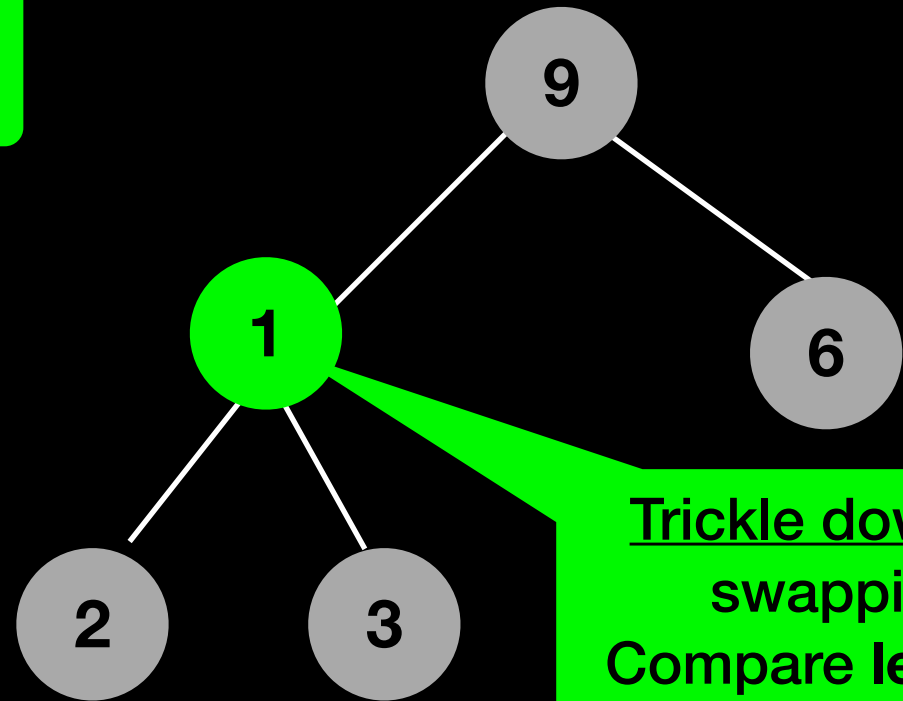
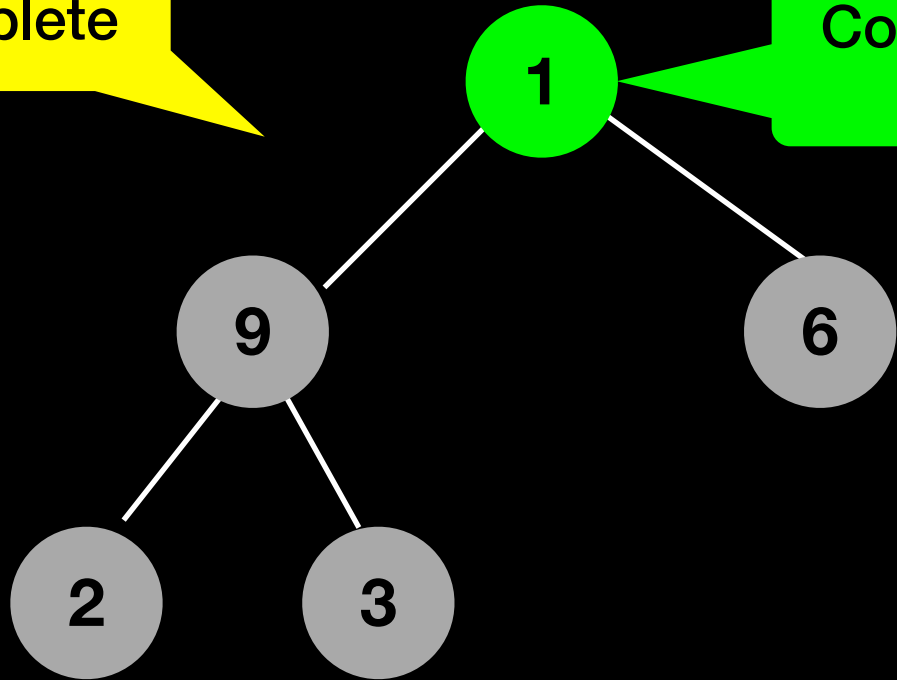
items_[i] left_child = items_[2 * i + 1]
items_[i] right_child = items_[2 * i + 2]
  
```

heapRebuild

# Remove

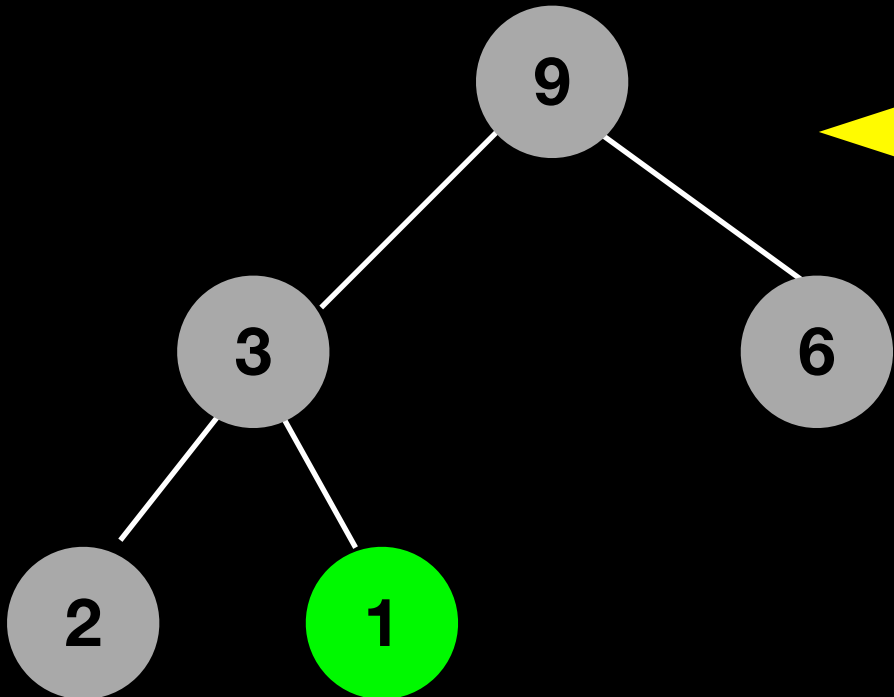
Complete

Trickle down by swapping  
Compare left and right



Trickle down by swapping  
Compare left and right

Heap!

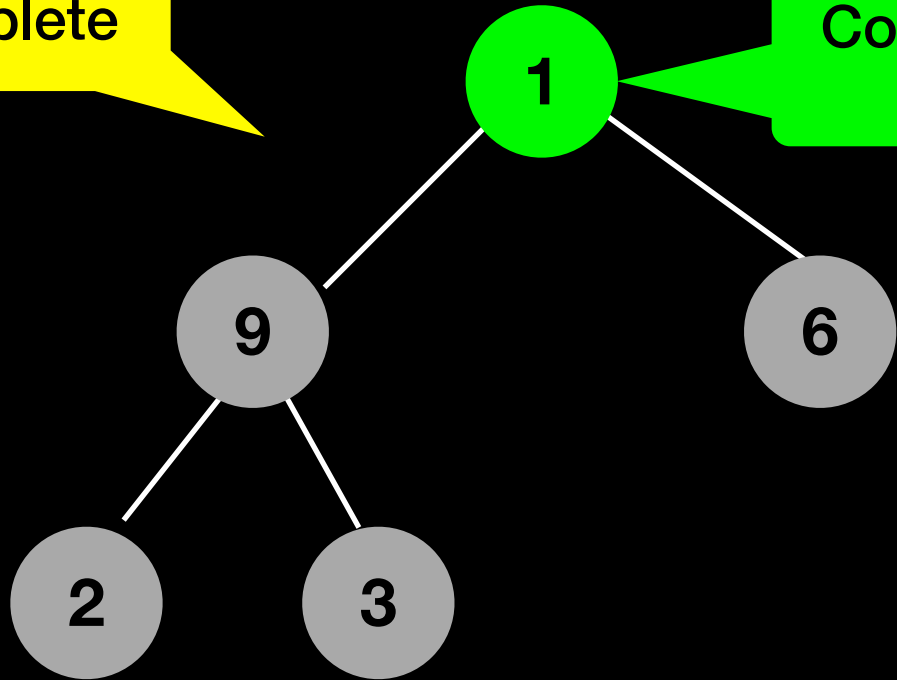


1	9	6	2	3
1	9	6	2	3
9	1	6	2	3
9	3	6	2	1

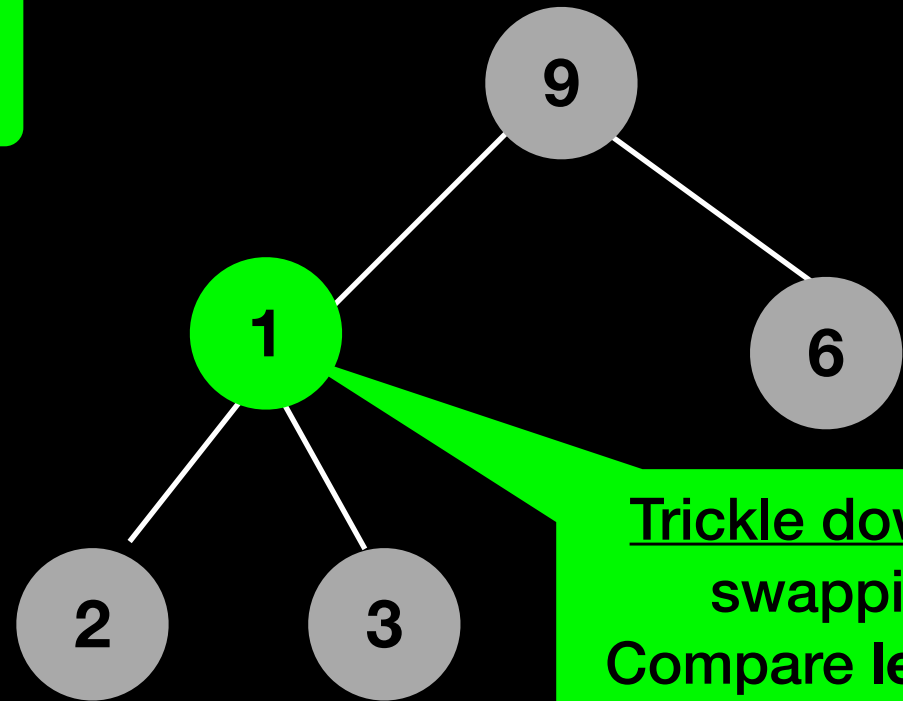
heapRebuild

# Remove

Complete

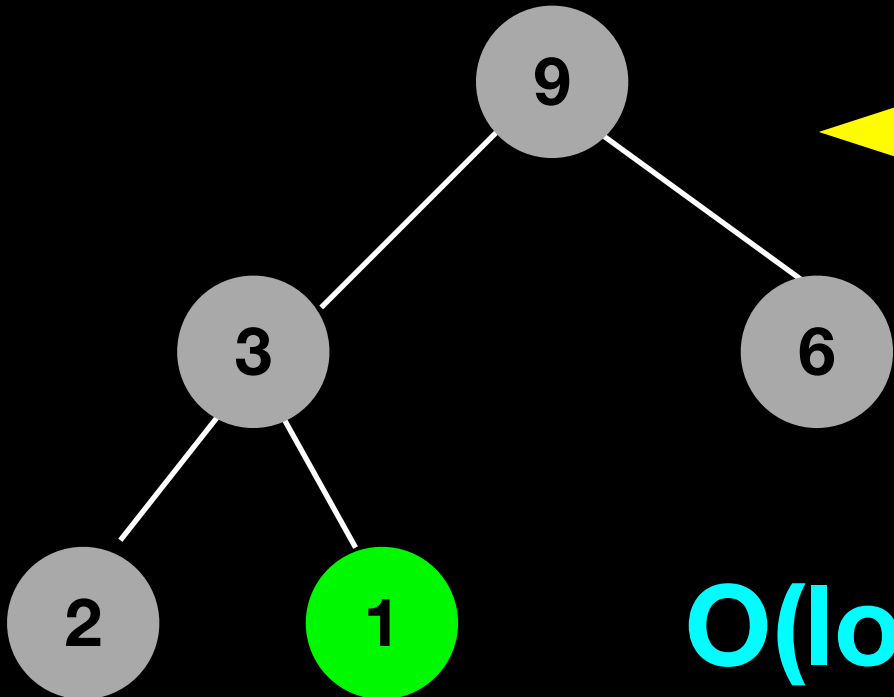


Trickle down by swapping  
Compare left and right



Trickle down by swapping  
Compare left and right

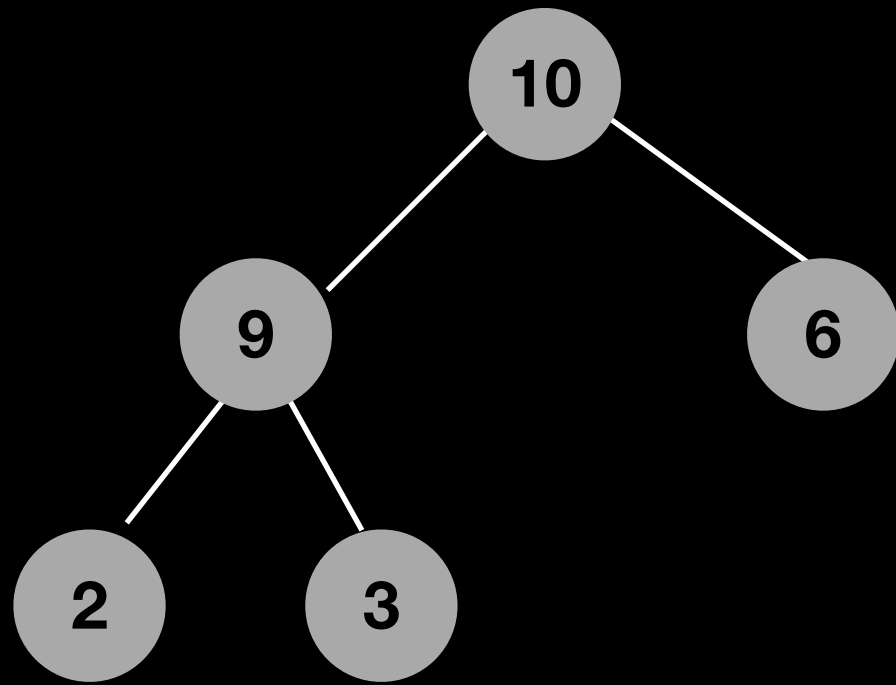
Heap!



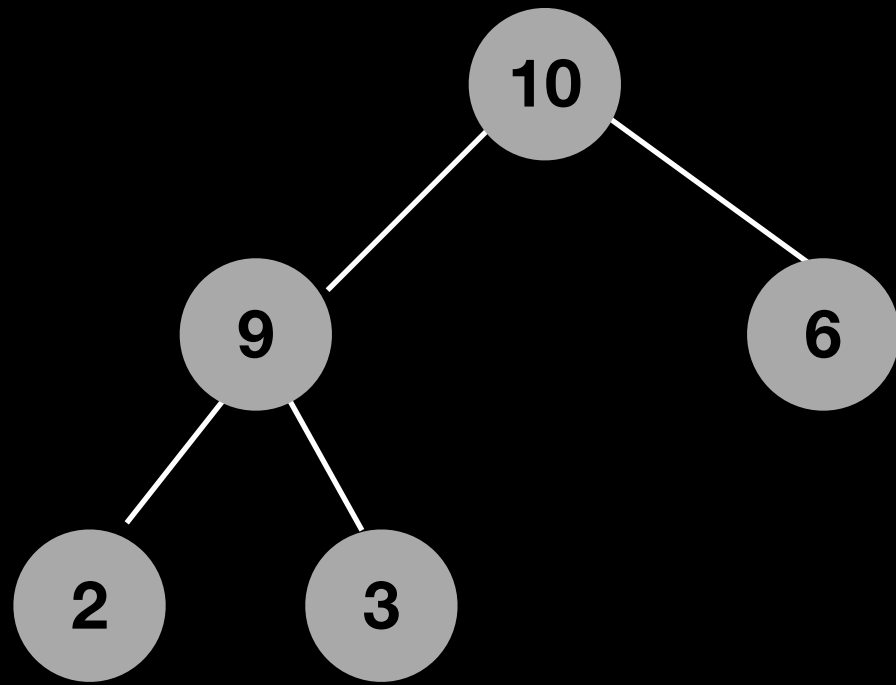
$O(\log n)$

1	9	6	2	3
1	9	6	2	3
9	1	6	2	3
9	3	6	2	1

# Add

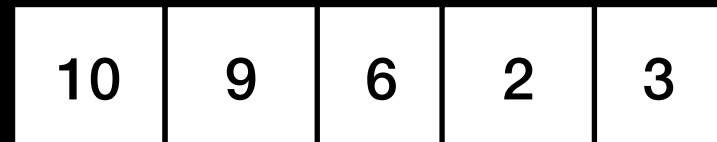


# Add



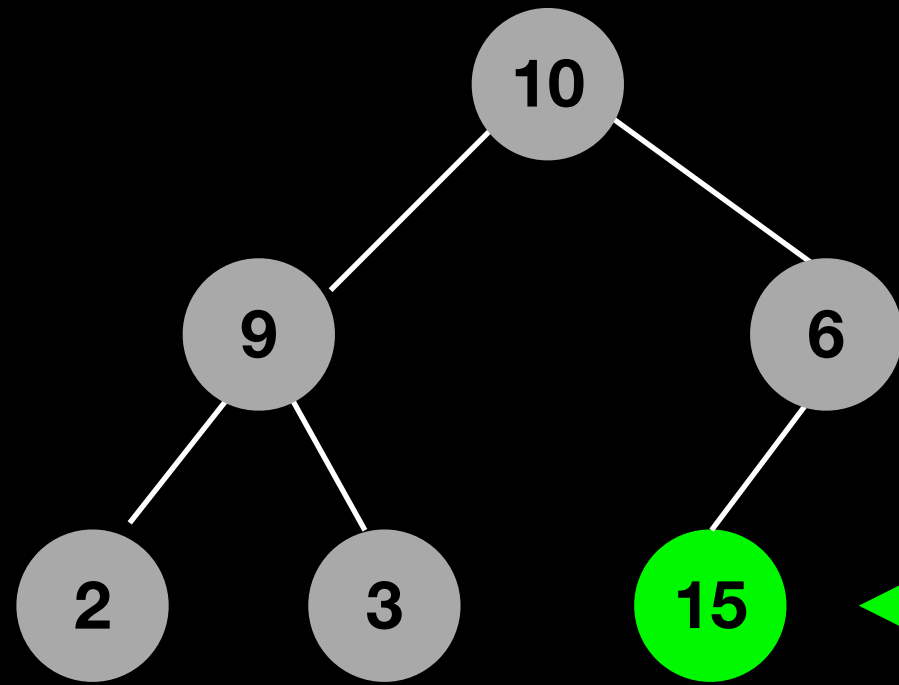
15

Where do we add?

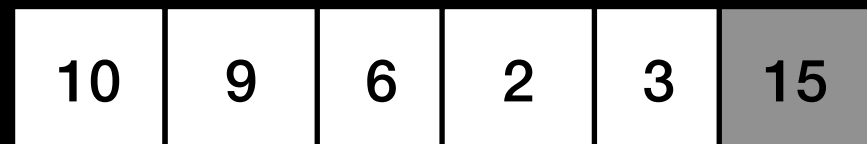




# Add

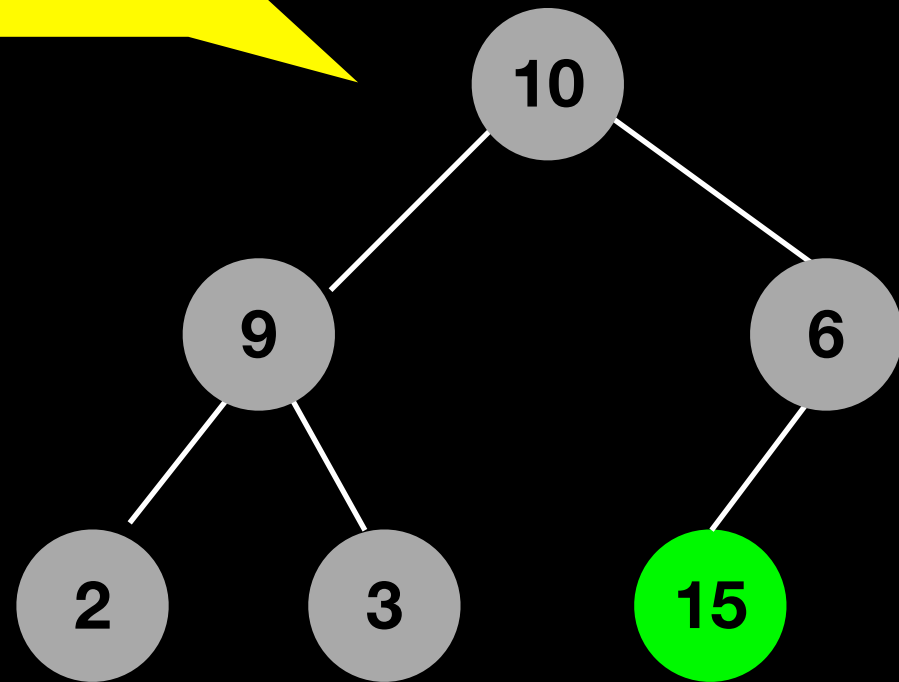


**Add here for complete tree**



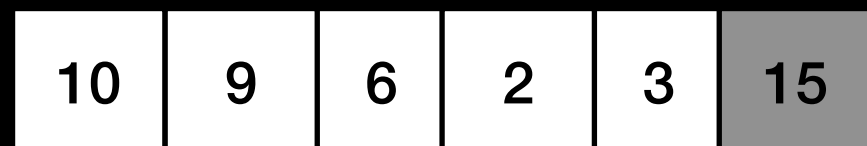
```
items_[i] left_child = items_[2 * i + 1]
```

Complete

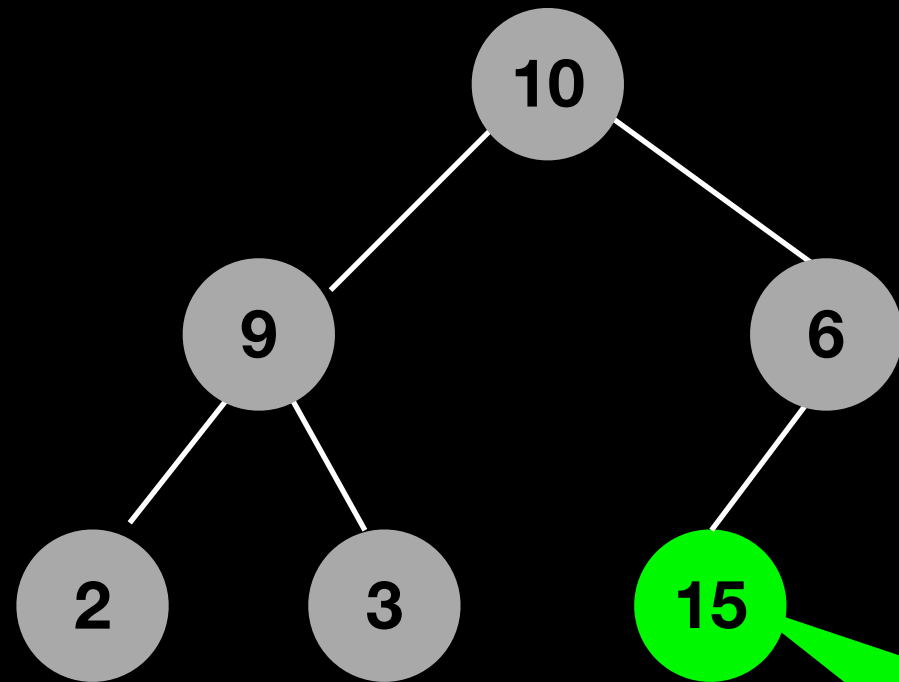


# Add

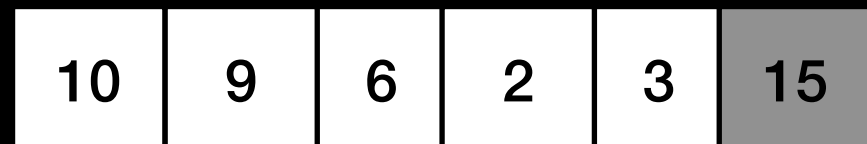
Not Heap



# Add

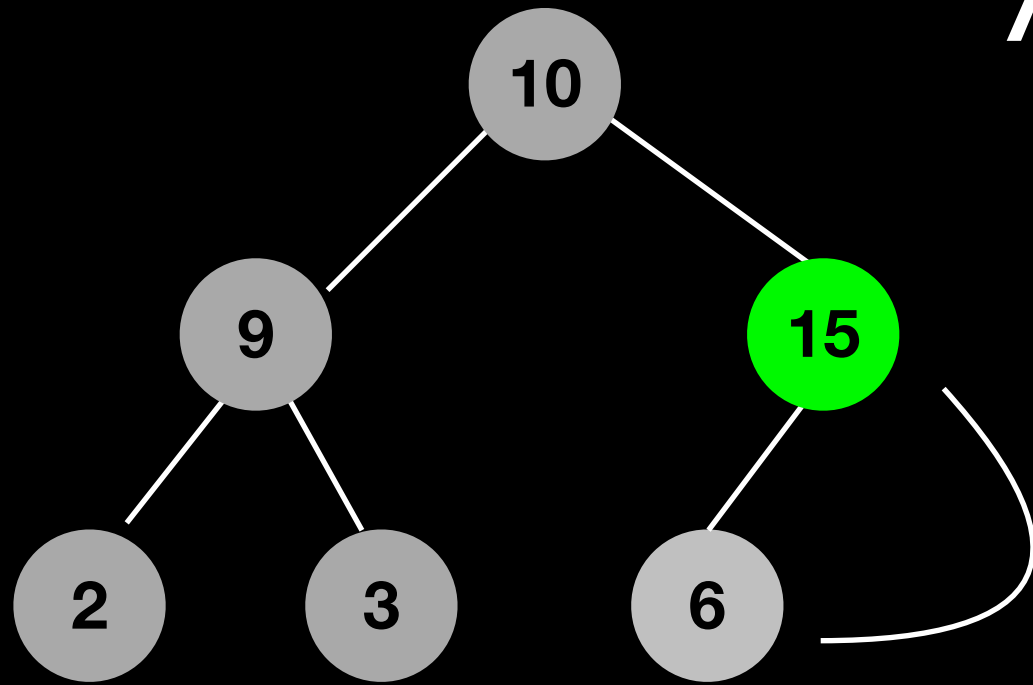


Bubble-up by  
swapping  
with parent



```
items_[i] parent = items_[(i-1)//2]
```

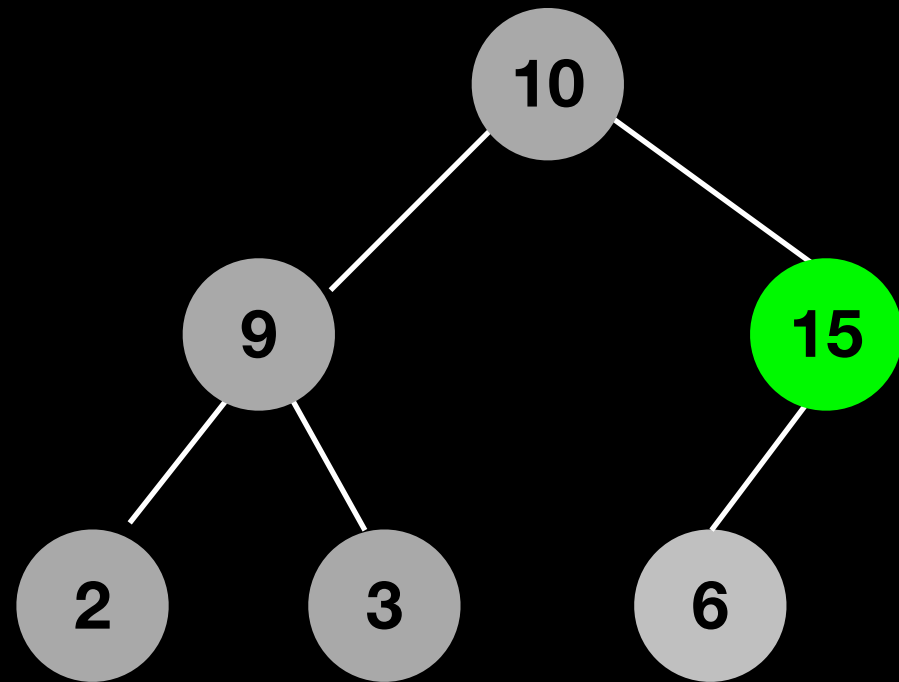
# Add



10	9	6	2	3	15
10	9	15	2	3	6

```
items_[i] parent = items_[(i-1)//2]
```

# Add

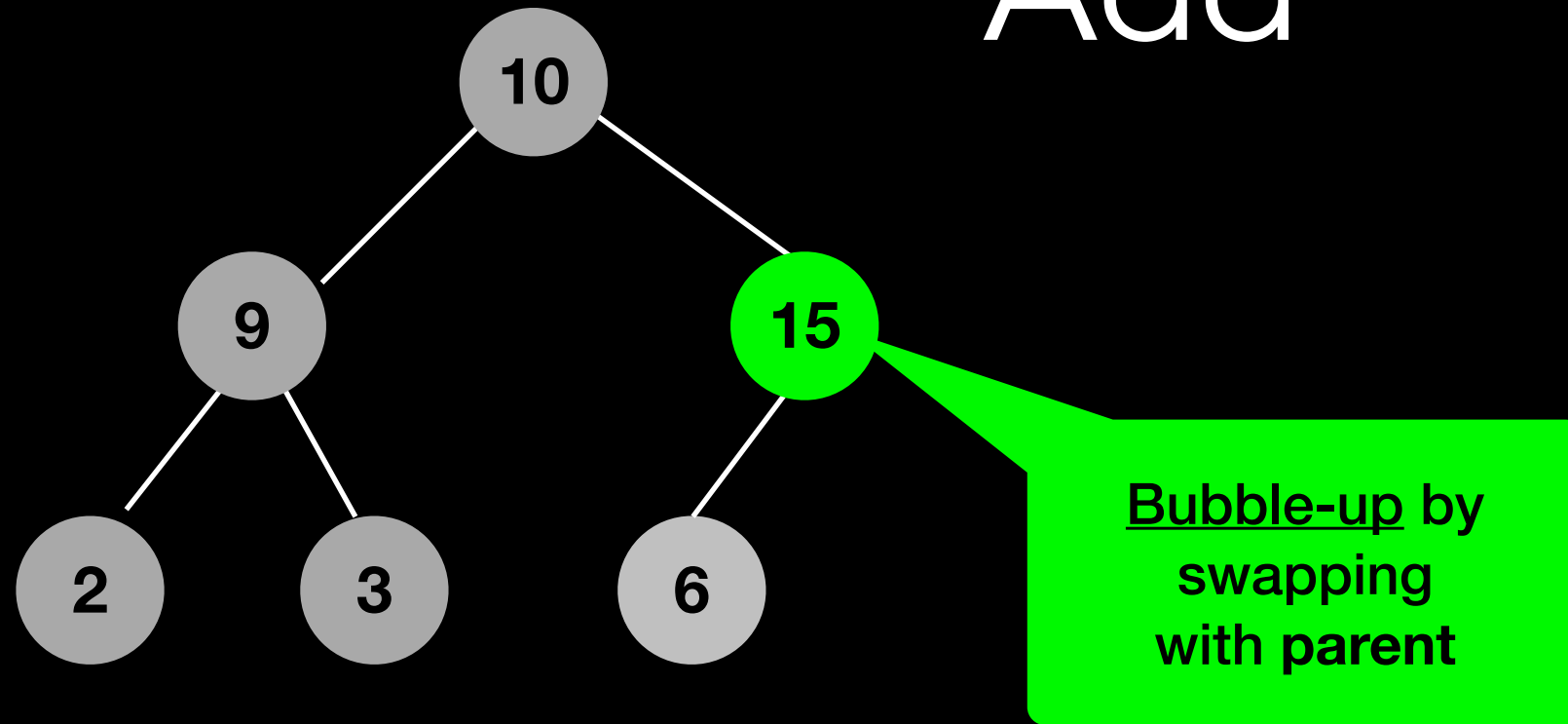


Not Heap

10	9	6	2	3	15
10	9	15	2	3	6

```
items_[i] parent = items_[(i-1)//2]
```

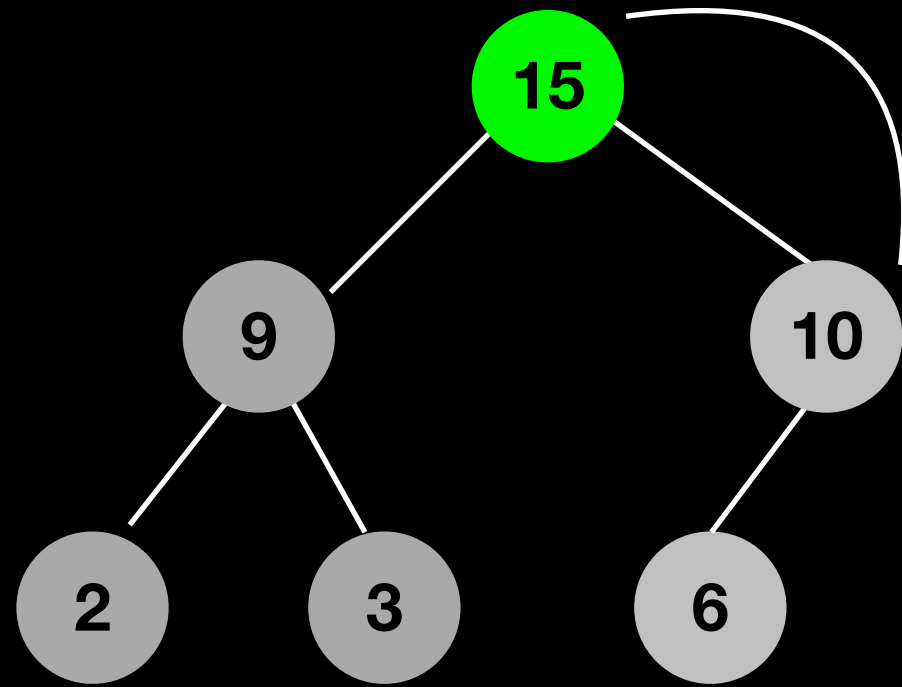
# Add



10	9	6	2	3	15
10	9	15	2	3	6

```
items_[i] parent = items_[(i-1)//2]
```

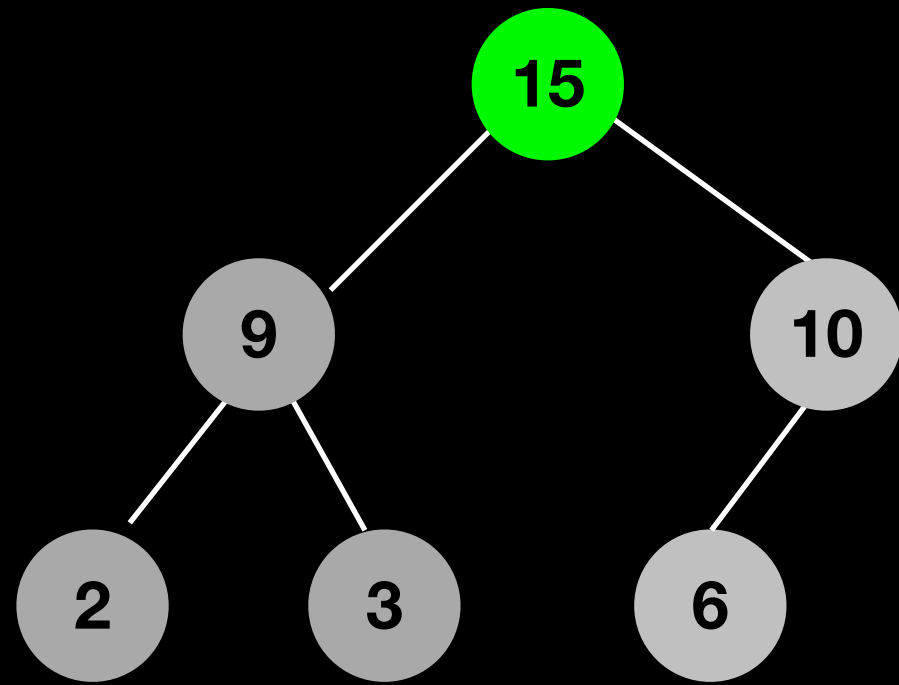
# Add



10	9	6	2	3	15
10	9	15	2	3	6
15	9	10	2	3	6

```
items_[i] parent = items_[(i-1)//2]
```

# Add



Heap!



$O(\log n)$

10	9	6	2	3	15
10	9	15	2	3	6
15	9	10	2	3	6

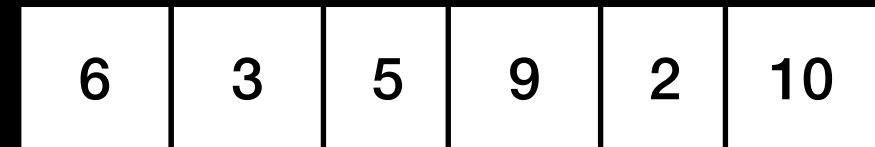
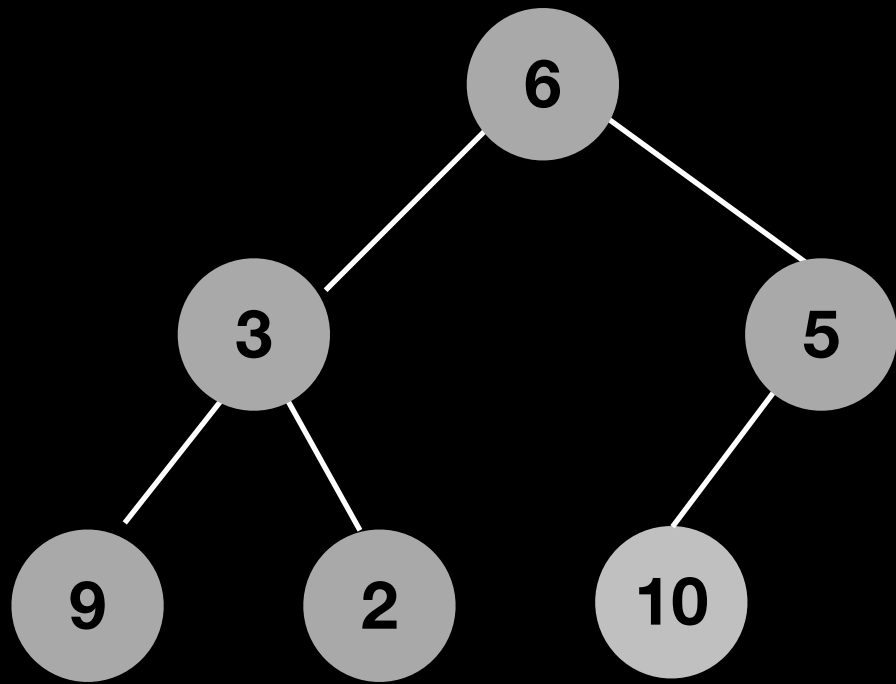
```
items_[i] parent = items_[(i-1)//2]
```



# heapCreate

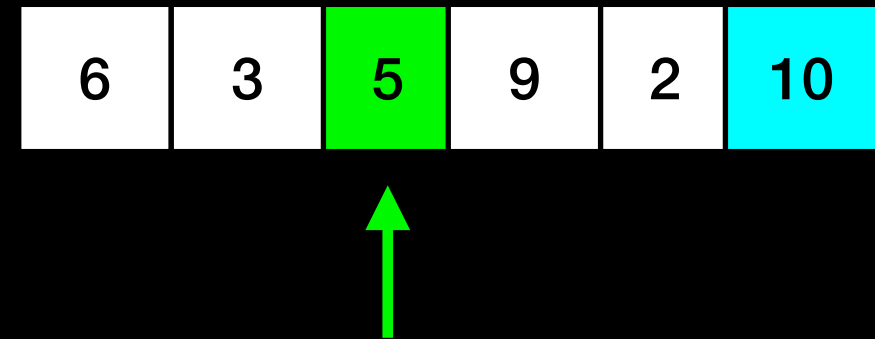
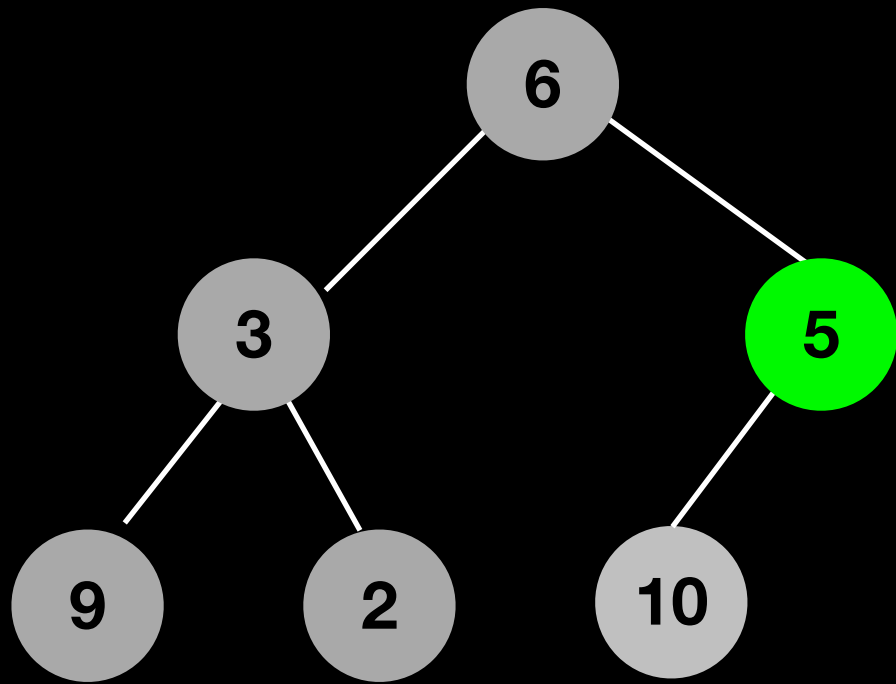
6	3	5	9	2	10
---	---	---	---	---	----

# heapCreate



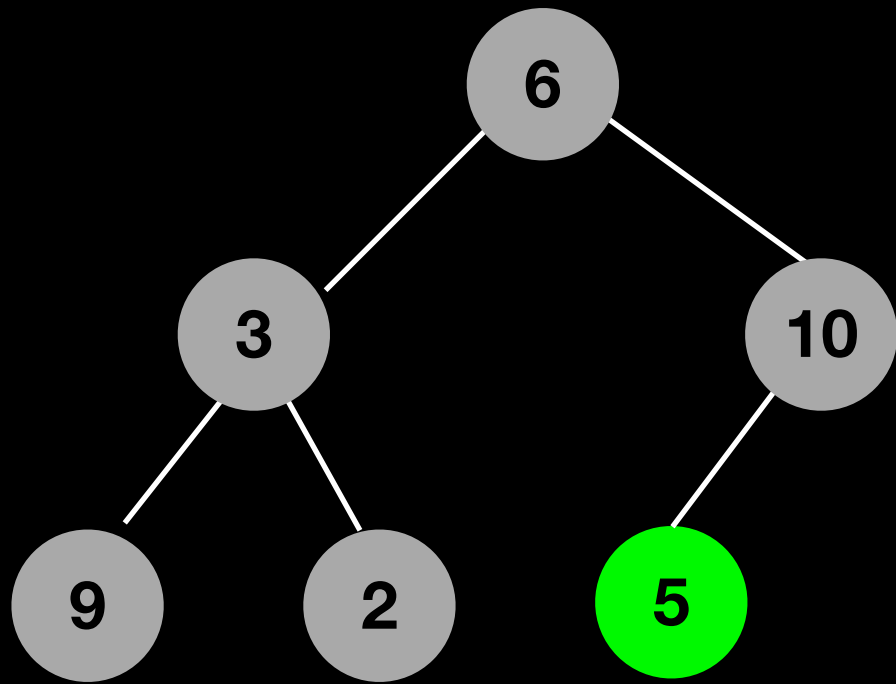
```
for(int i=(itemCount/2)-1; i >=0; i--)  
{  
    heapRebuild(i);  
}
```

# heapCreate

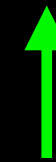


```
for(int i=(itemCount/2)-1; i >=0; i--)  
{  
    heapRebuild(i);  
}
```

# heapCreate

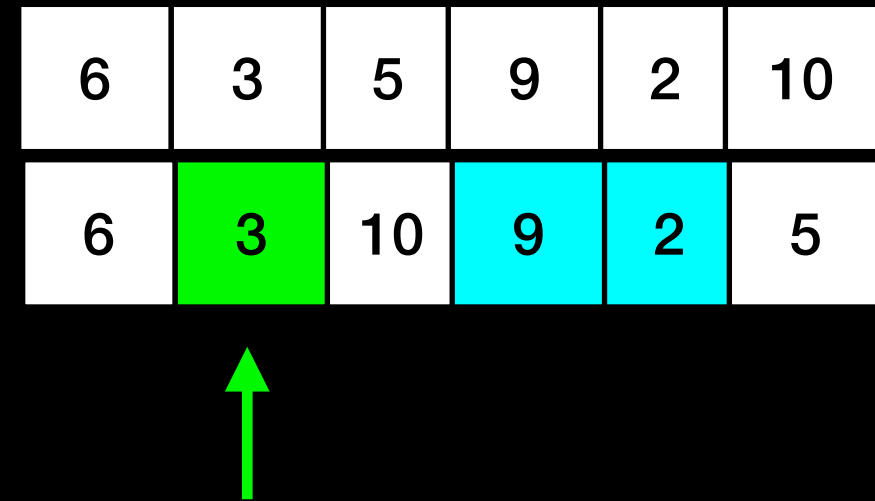
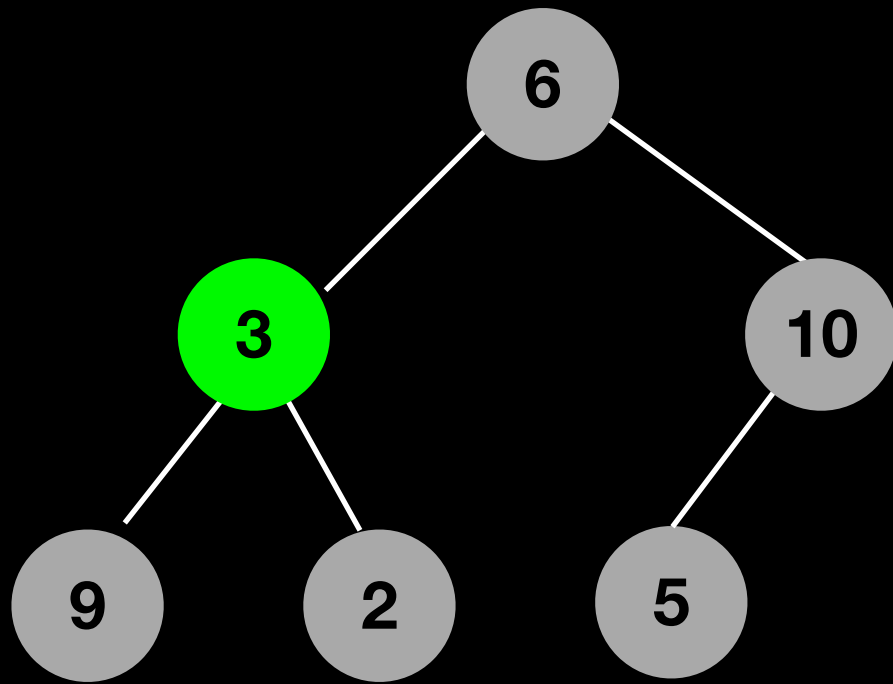


6	3	5	9	2	10
6	3	10	9	2	5



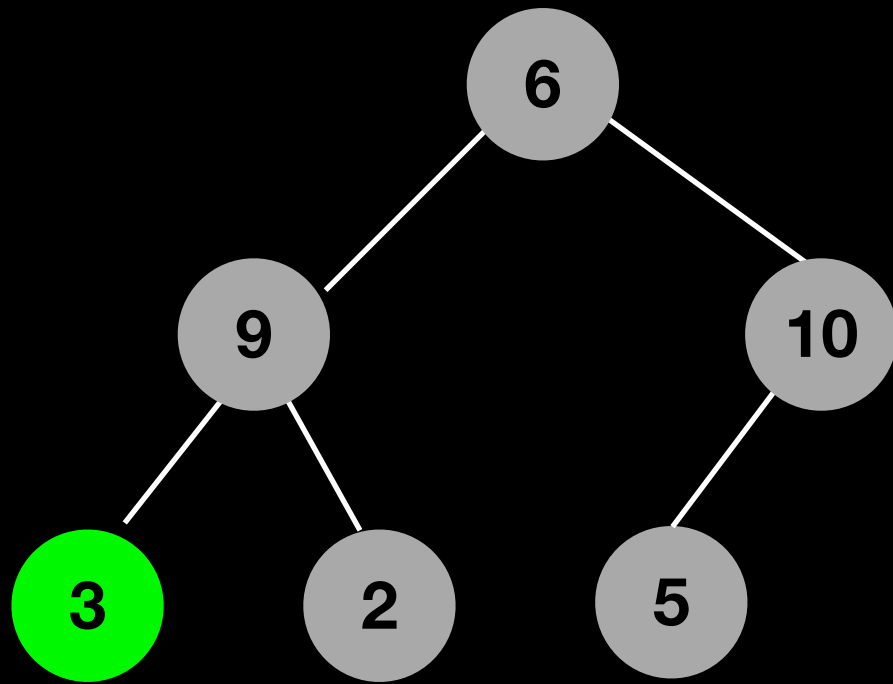
```
for(int i=(itemCount/2)-1; i >=0; i--)  
{  
    heapRebuild(i);  
}
```

# heapCreate

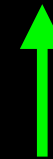


```
for(int i=(itemCount/2)-1; i >=0; i--)  
{  
    heapRebuild(i);  
}
```

# heapCreate

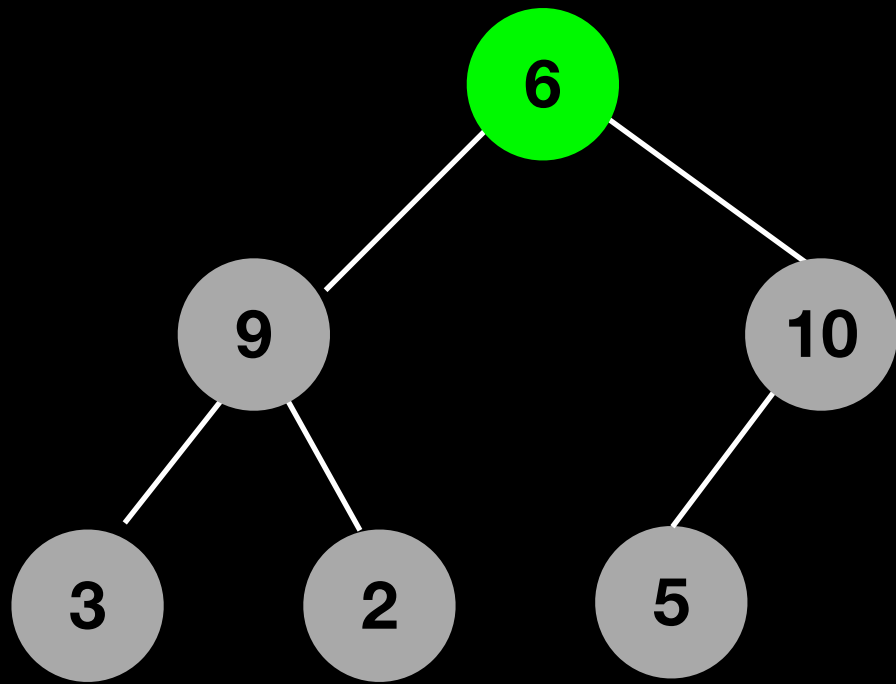


6	3	5	9	2	10
6	3	10	9	2	5
6	9	10	3	2	5

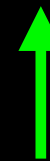


```
for(int i=(itemCount/2)-1; i >=0; i--)  
{  
    heapRebuild(i);  
}
```

# heapCreate

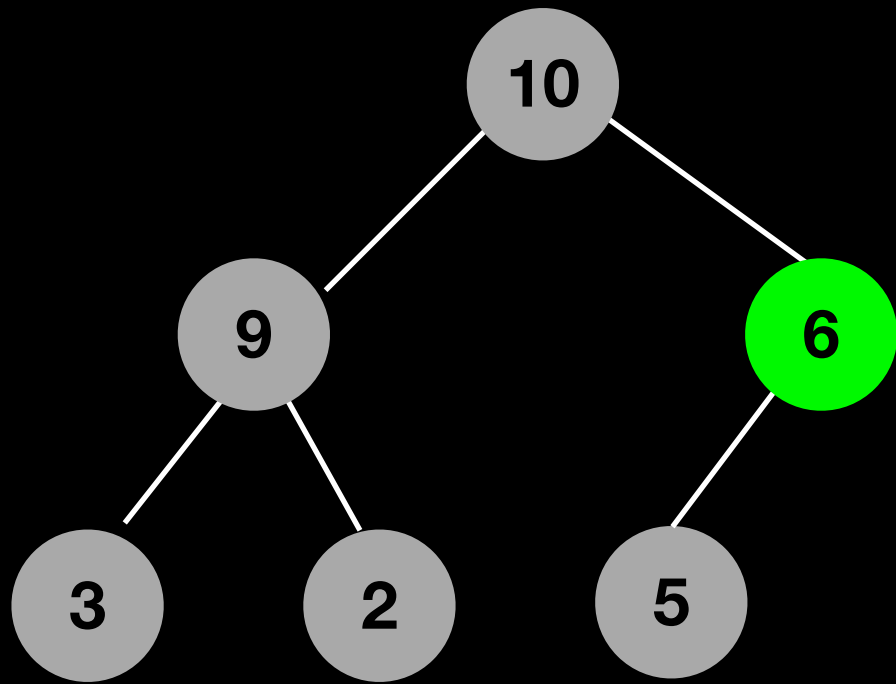


6	3	5	9	2	10
6	3	10	9	2	5
6	9	10	3	2	5

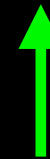


```
for(int i=(itemCount/2)-1; i >=0; i--)  
{  
    heapRebuild(i);  
}
```

# heapCreate



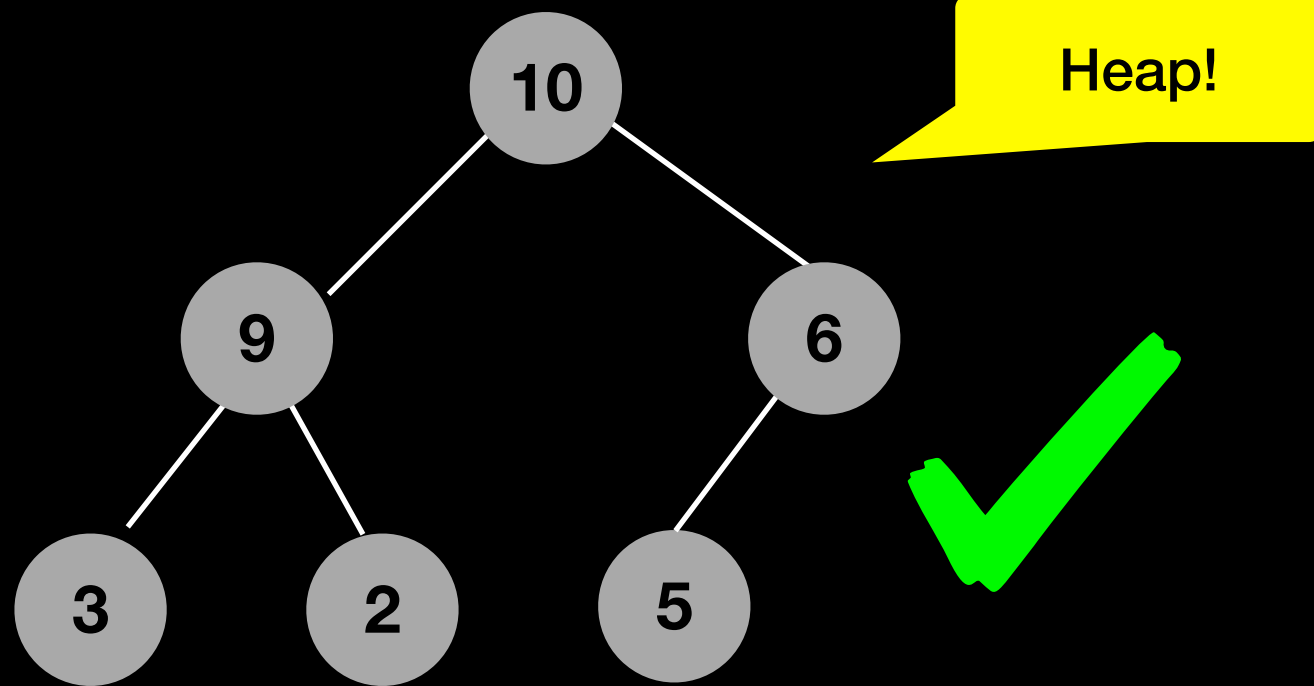
6	3	5	9	2	10
6	3	10	9	2	5
10	9	6	3	2	5



```
for(int i=(itemCount/2)-1; i >=0; i--)  
{  
    heapRebuild(i);  
}
```



# heapCreate



6	3	5	9	2	10
6	3	10	9	2	5
10	9	6	3	2	5

**n/2 heapRebuild**

```
for(int i=(itemCount/2)-1; i >=0; i--)  
{  
    heapRebuild(i);  
}
```

# Heapsort

# Heapsort

Given an unsorted array:

9	3	10	5	2	6
---	---	----	---	---	---

- `heapCreate`

- $\text{last} = n - 2$

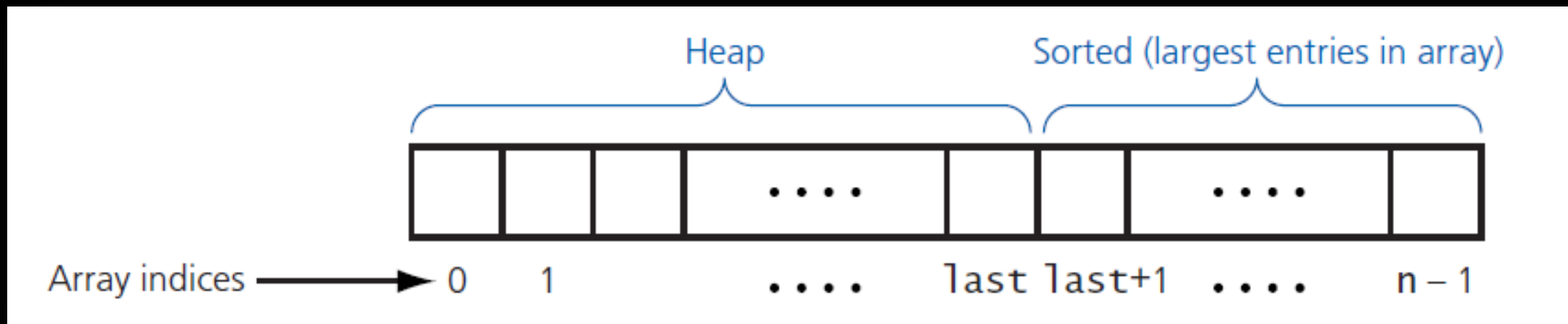
- repeat:

  - swap `items[0]` with `items[last+1]`

  - $\text{last}--$

  - `heapRebuild`

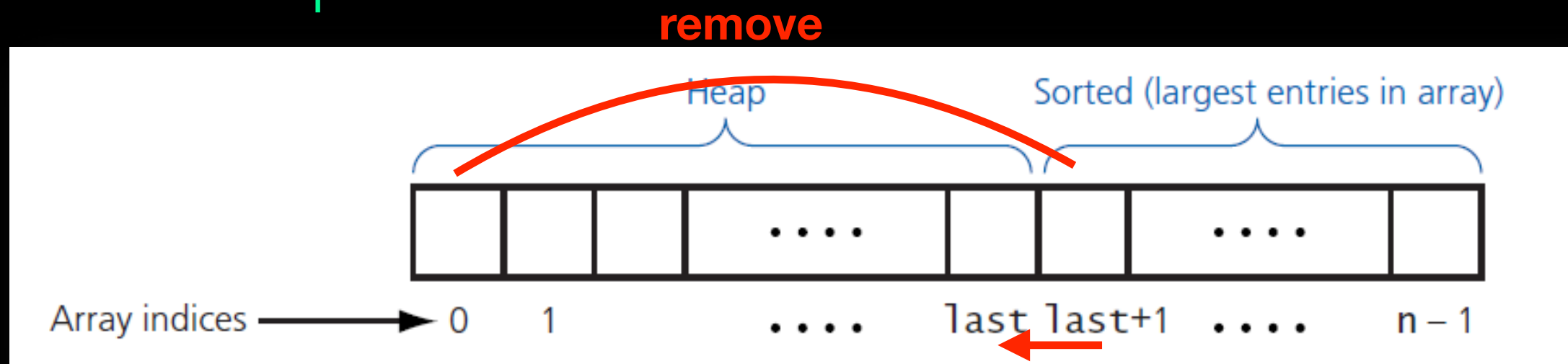
10	9	6	3	2	5
----	---	---	---	---	---



# Heapsort

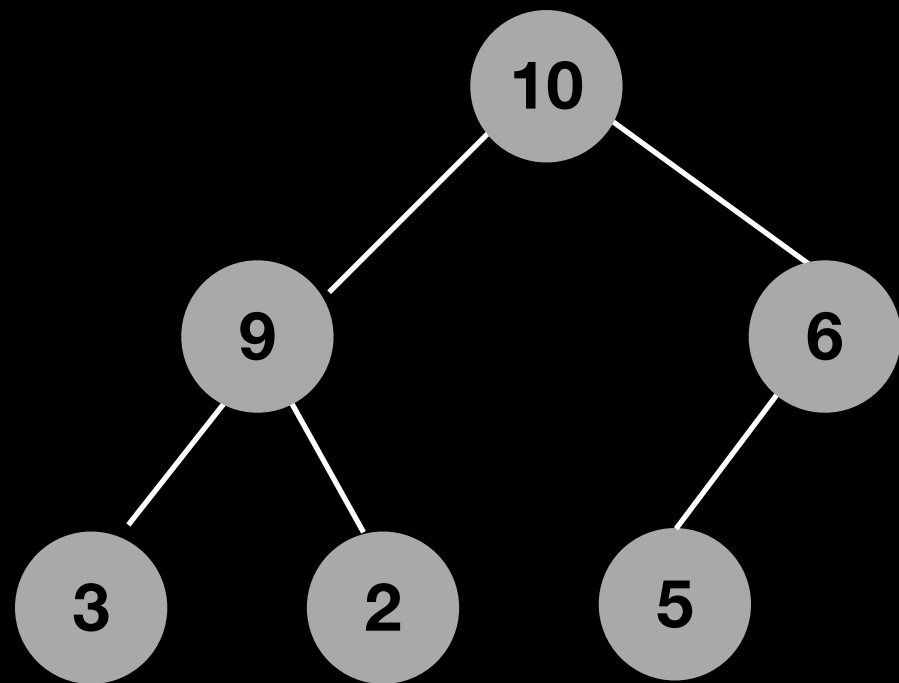
Given an unsorted array:

- **heapCreate**
- $\text{last} = n - 2$
- repeat:
  - swap items[0] with items[last+1]
  - last--
  - **heapRebuild**

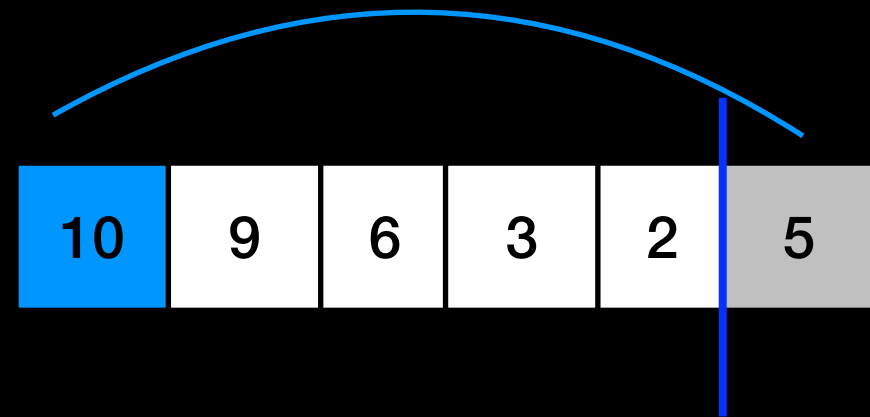
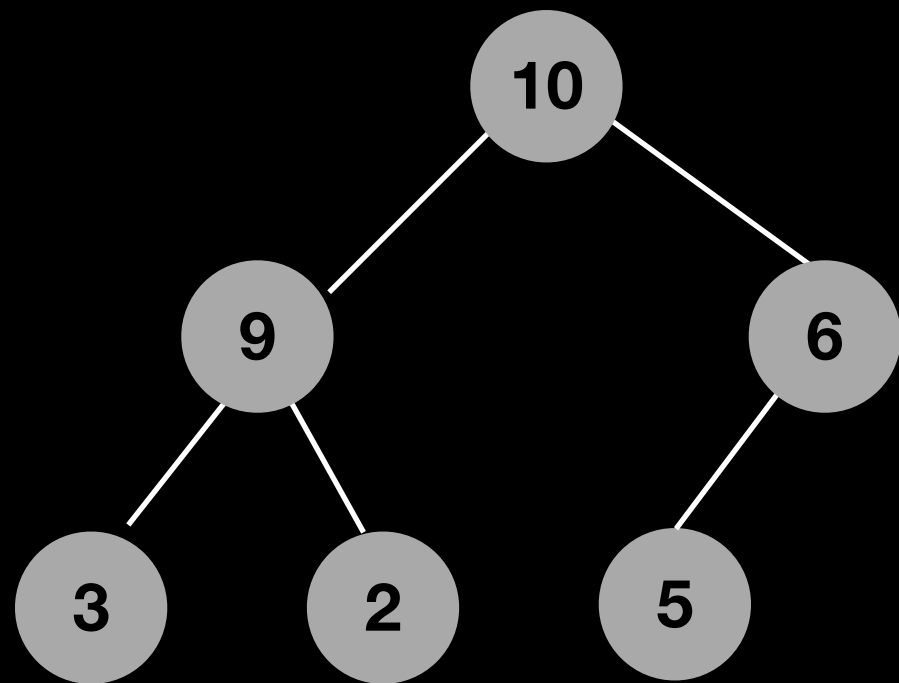


# Heapsort

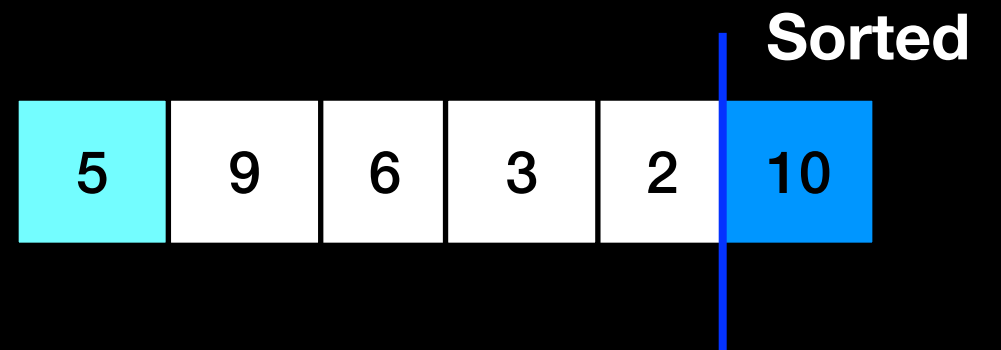
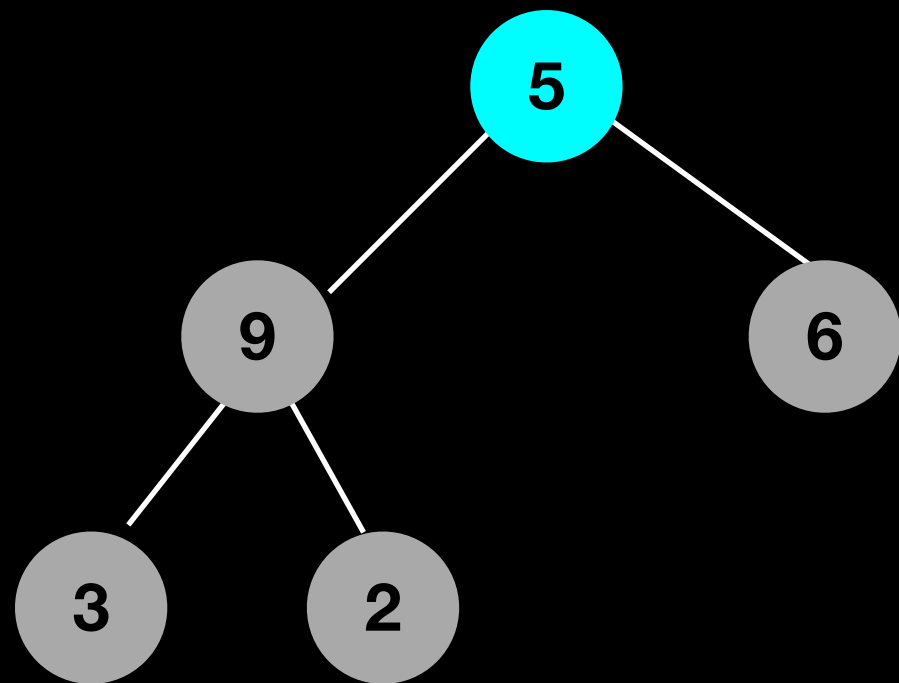
10	9	6	3	2	5
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# Heapsort

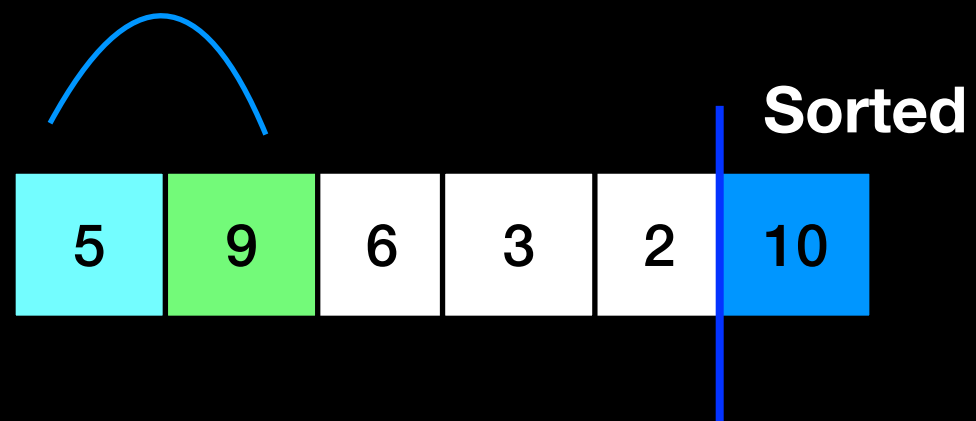
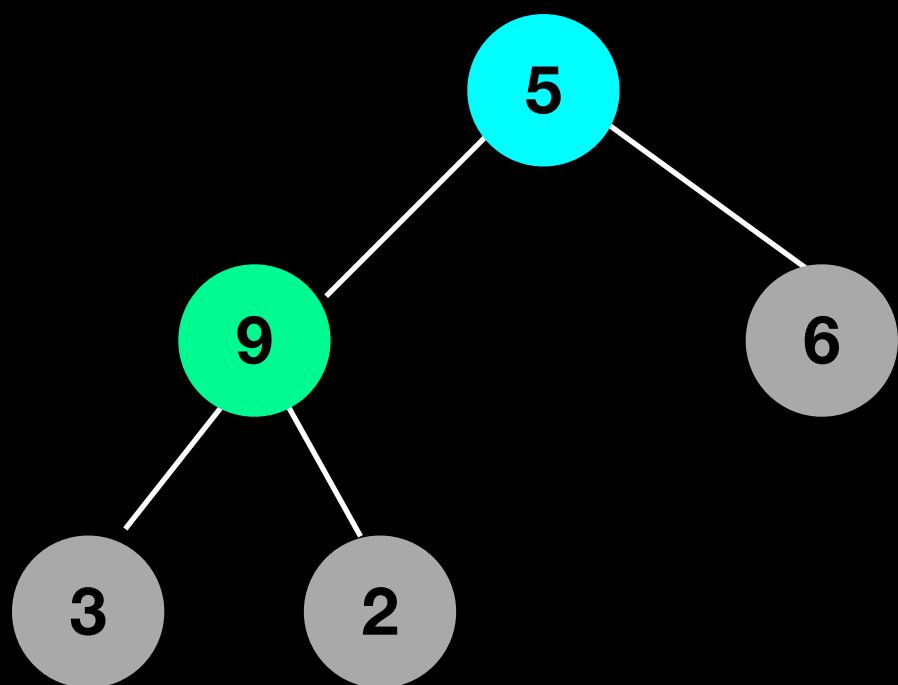


# Heapsort



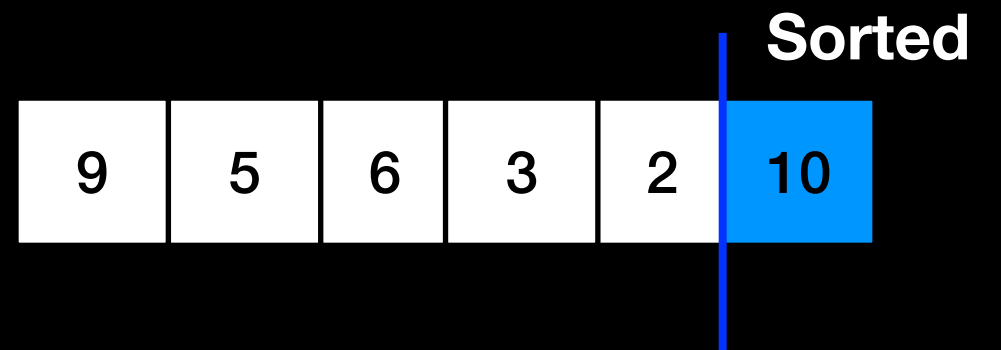
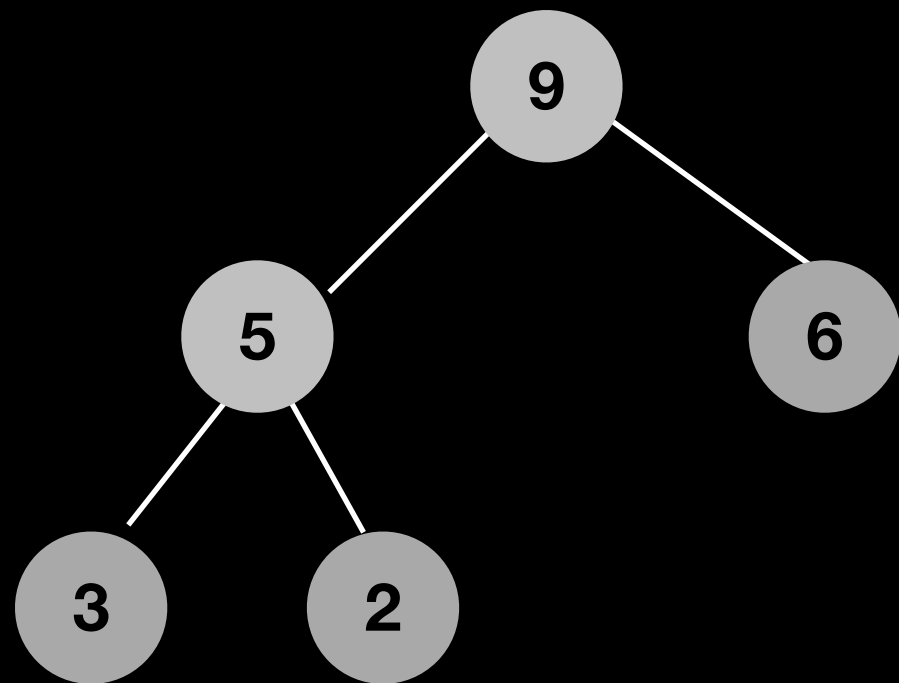
heapRebuild

# Heapsort

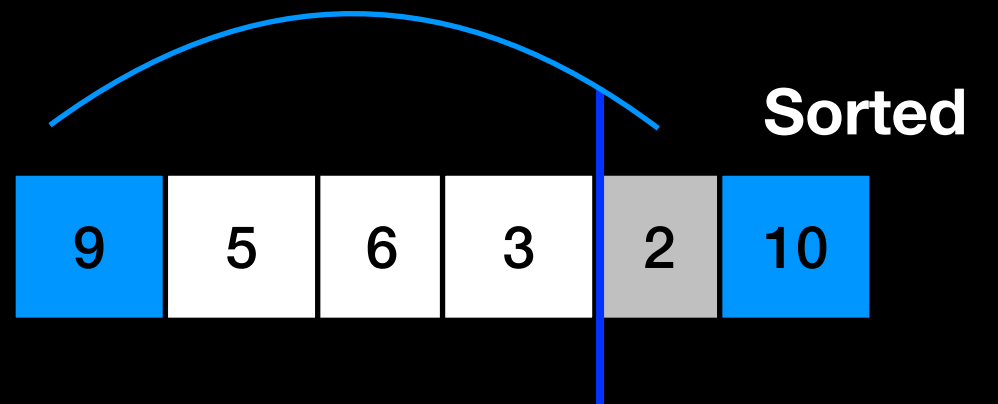
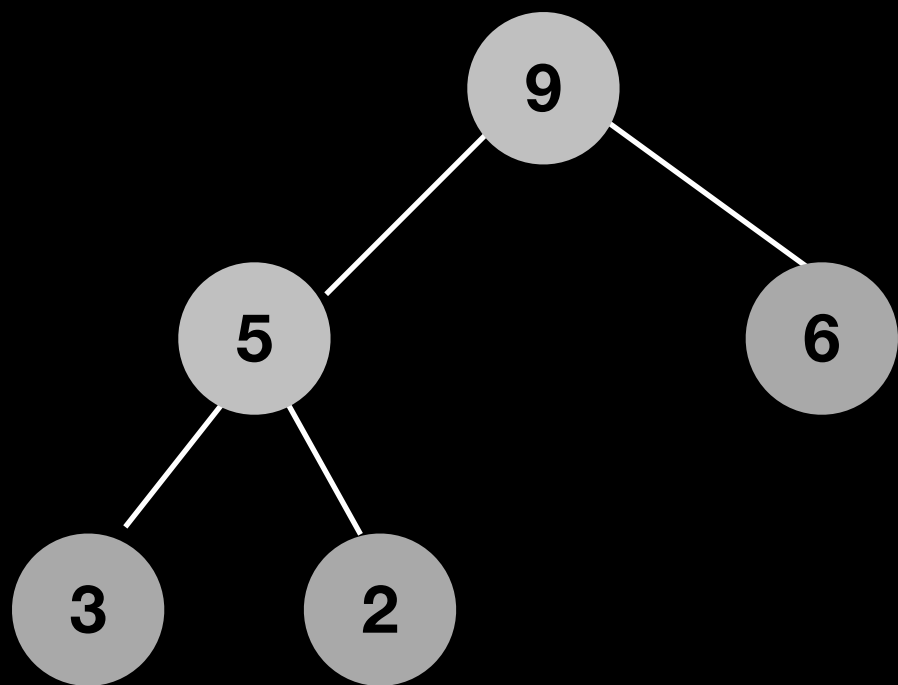




# Heapsort

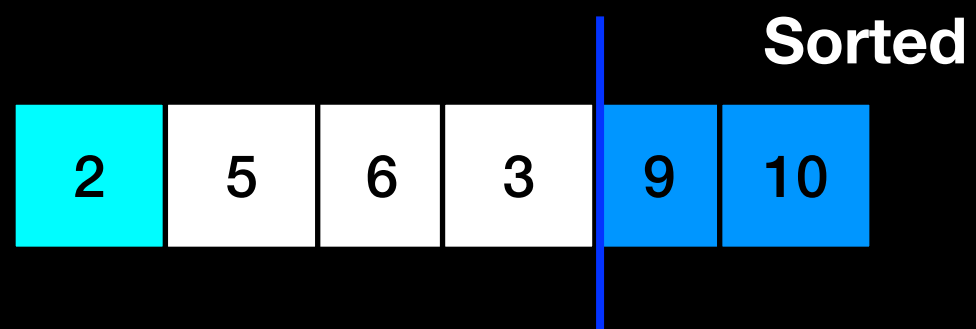
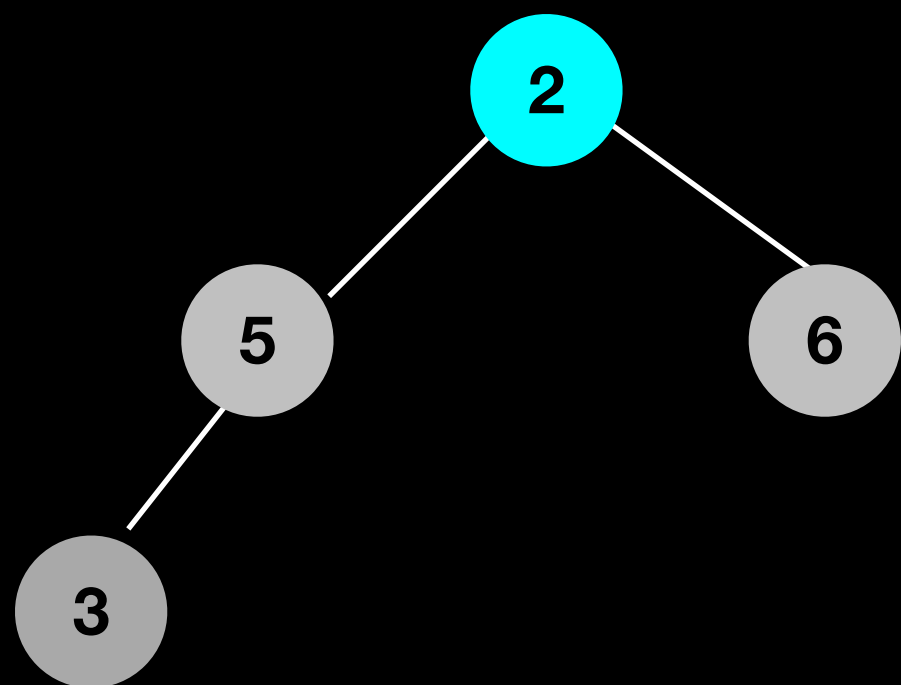


# Heapsort



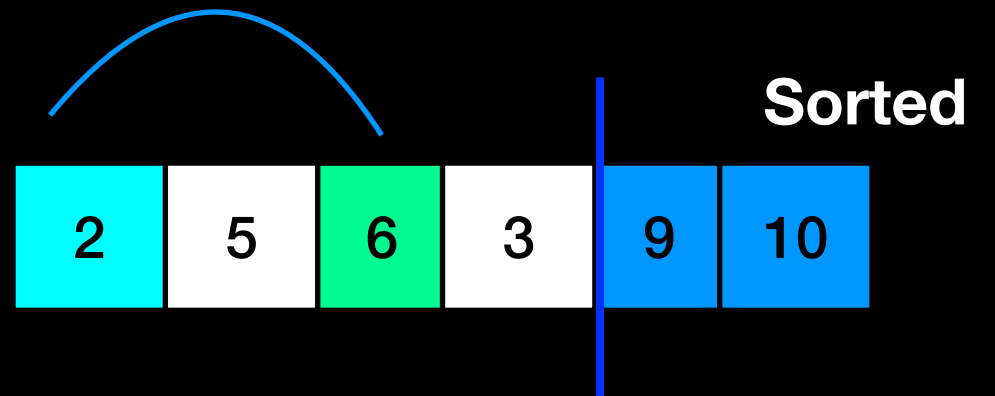
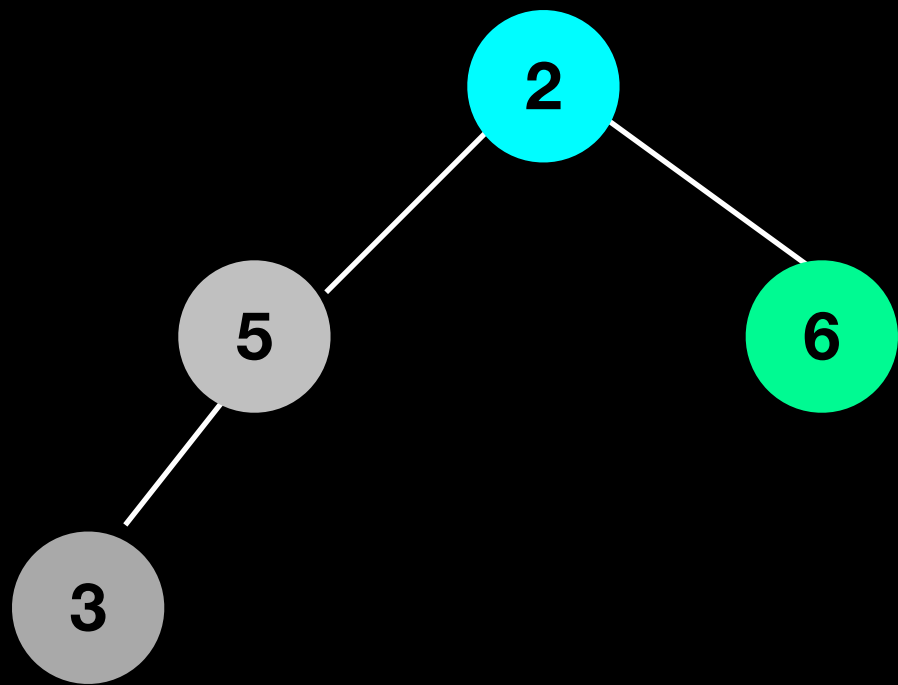
heapRebuild

# Heapsort

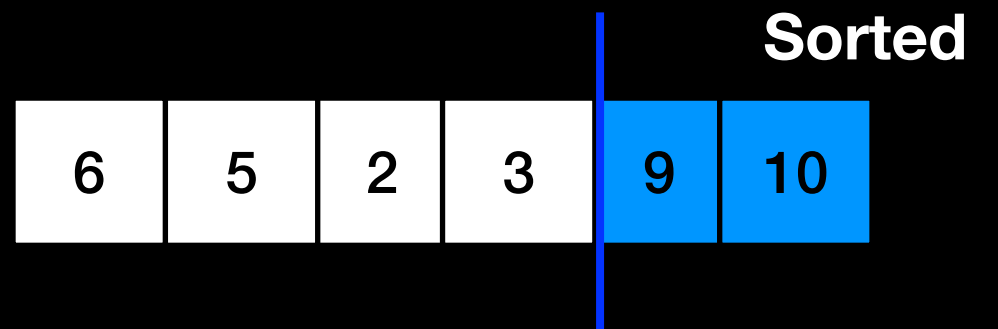
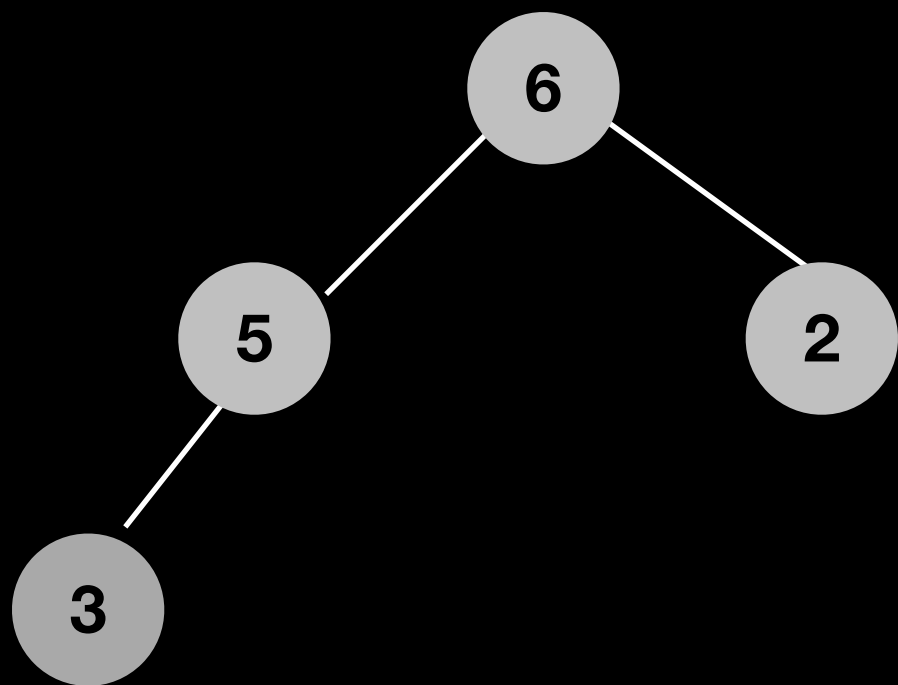


heapRebuild

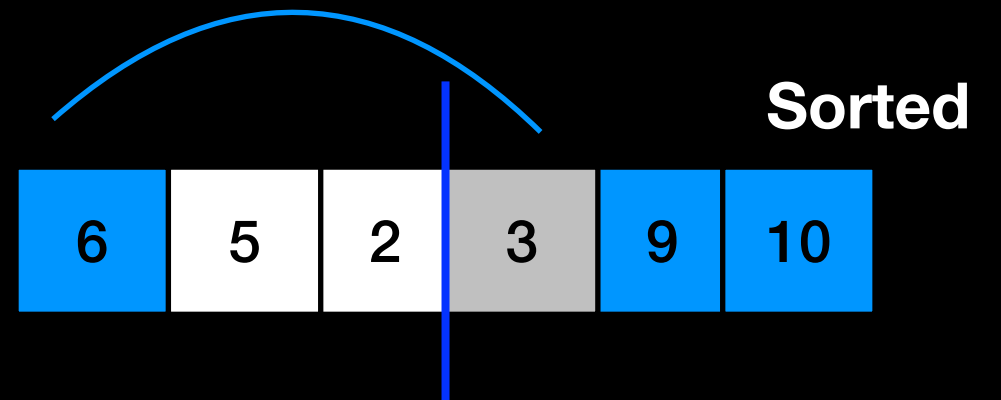
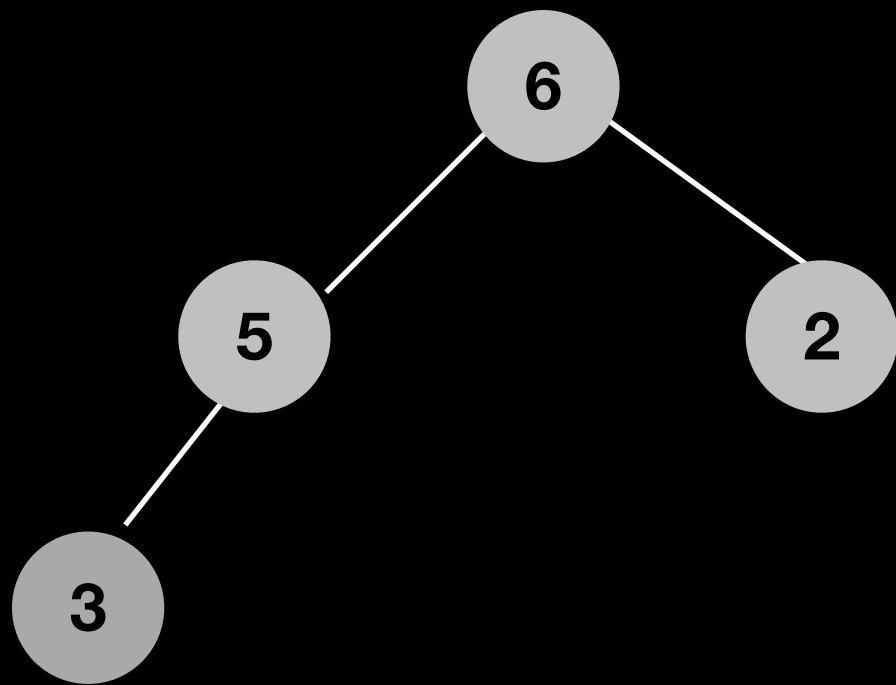
# Heapsort



# Heapsort

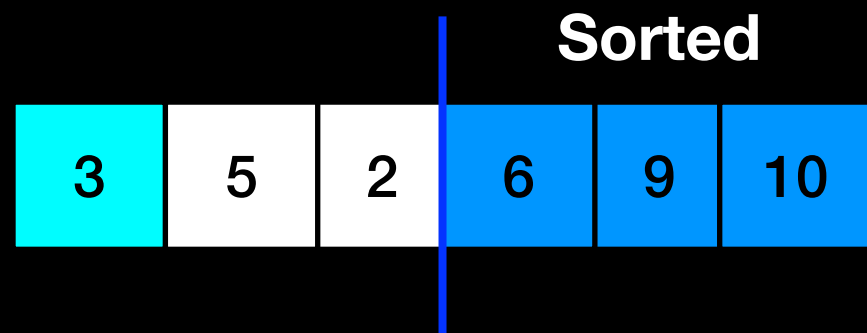
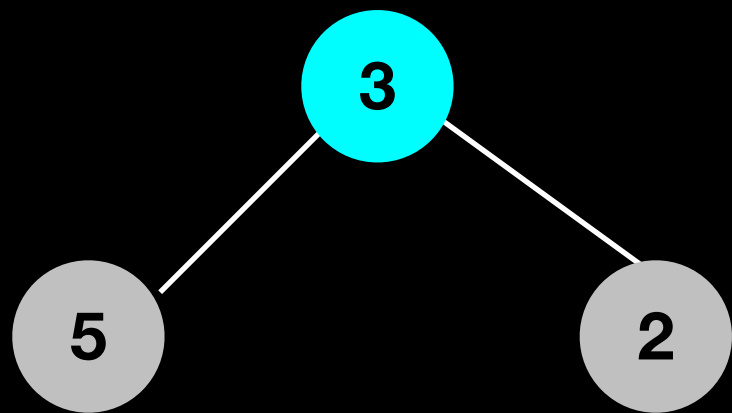


# Heapsort



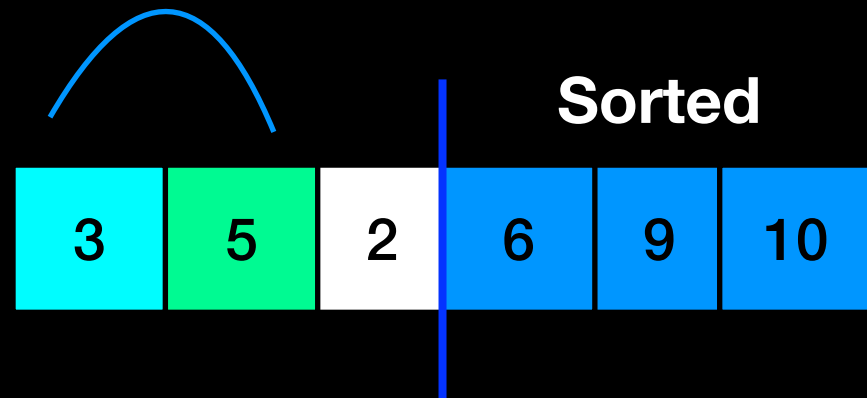
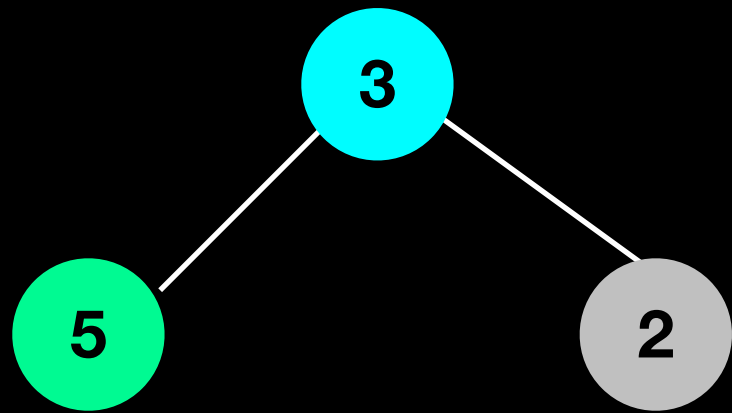
heapRebuild

# Heapsort



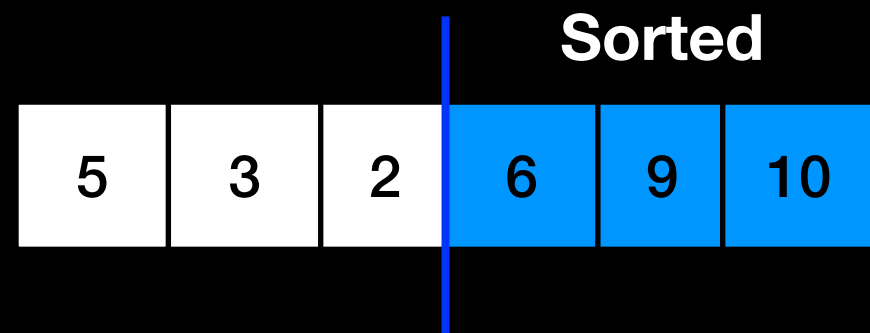
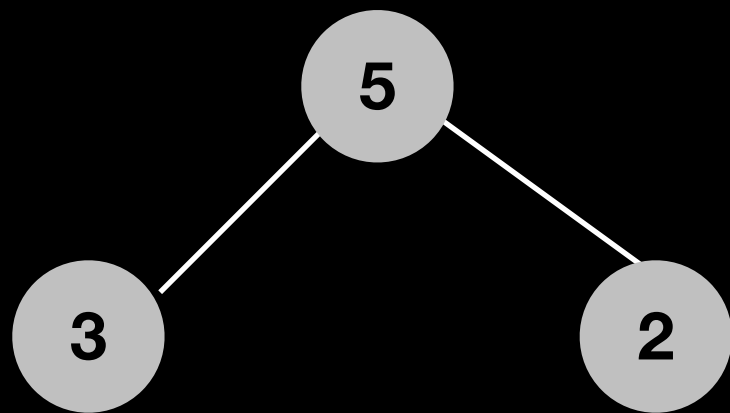
heapRebuild

# Heapsort

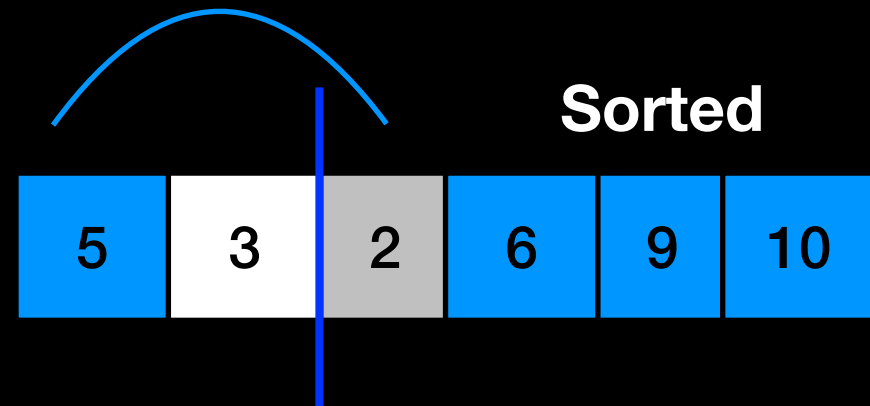
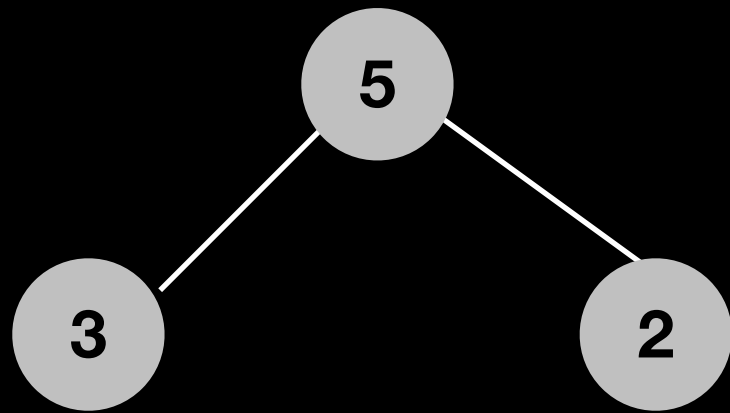




# Heapsort

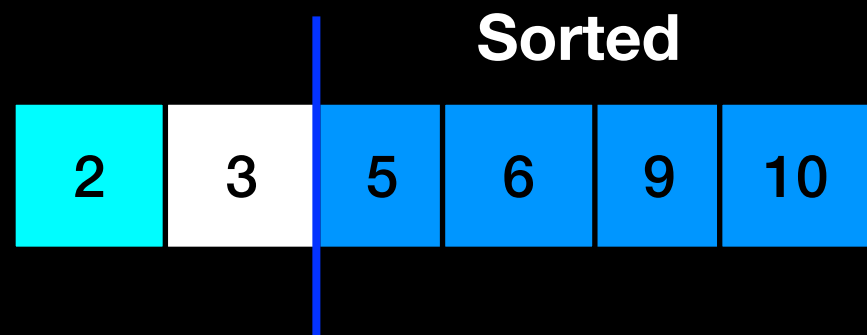
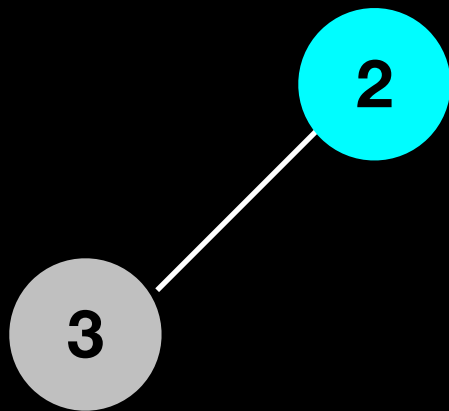


# Heapsort



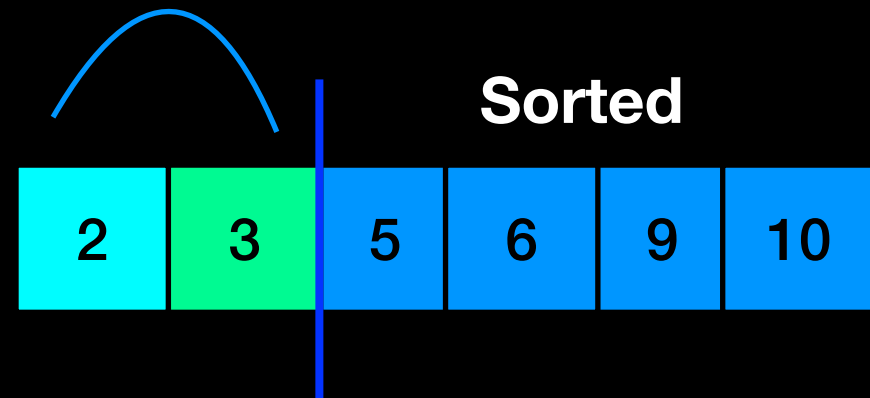
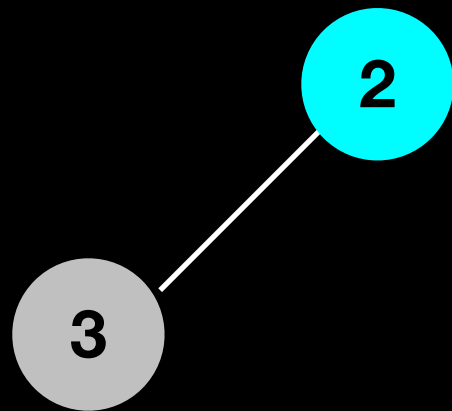
heapRebuild

# Heapsort

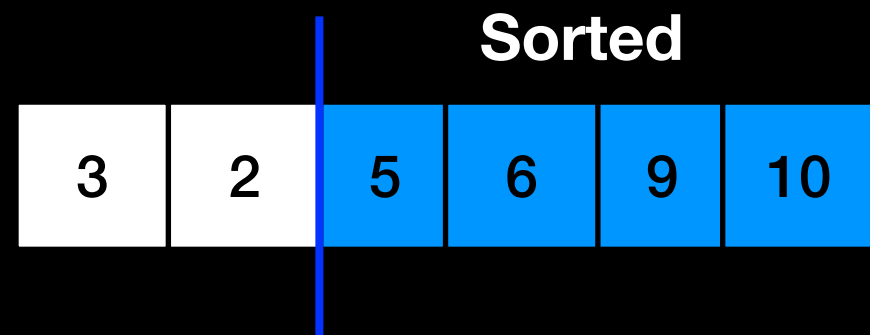
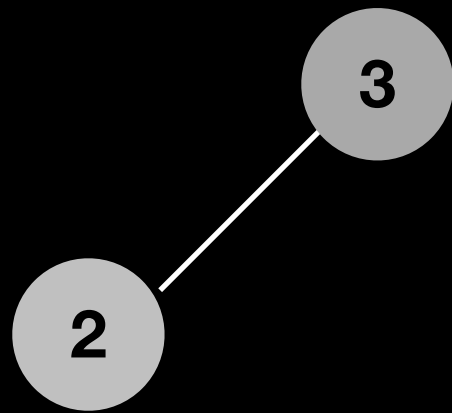


heapRebuild

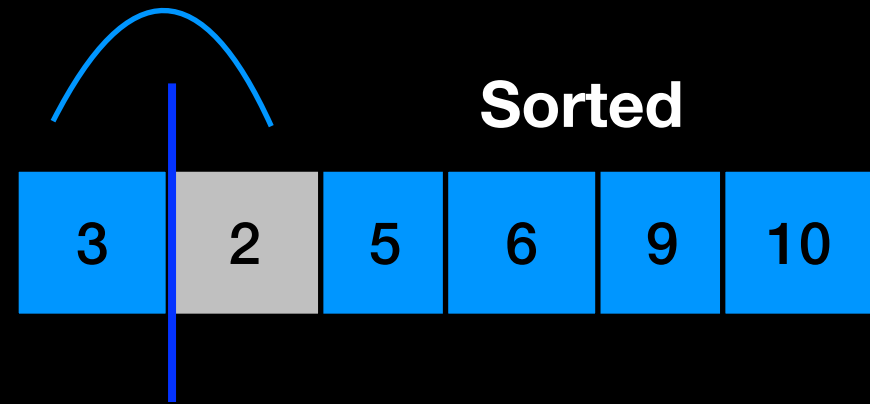
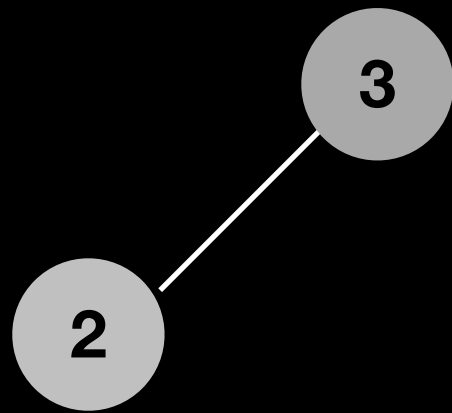
# Heapsort



# Heapsort

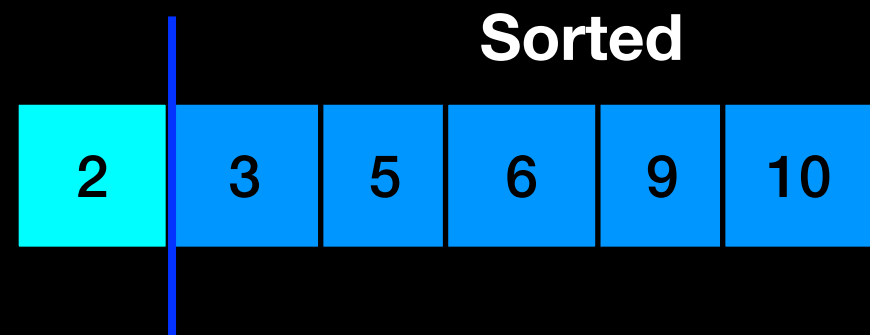


# Heapsort



# Heapsort

2



# Heapsort

Sorted

2	3	5	6	9	10
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# Heapsort Analysis

1. heapCreate ->  $O(n \log n)$

2. heapRebuild ->  $O(\log n)$  repeated for each of the  $n$  sorted items

$$O(n \log n) + O(n \log n) = \boxed{O(n \log n)}$$

Like MergeSort but no extra space needed!